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## Study of circular code motifs in nucleic acid sequences

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## Étude des motifs de code circulaire dans les séquences d'acides nucléiques

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#### Abstract

Le travail effectué dans cette thèse présente une nouvelle approche de la théorie du code circulaire dans les gènes qui a été initiée en 1996. Cette approche consiste à analyser les motifs construits à partir de ce code circulaire. Ces motifs particuliers sont appelés motifs de code circulaire. Ainsi, nous avons développé des algorithmes de recherche pour localiser les motifs de code circulaire dans les séquences d'acides nucléiques afin de leurs trouver une signification bioinformatique. En effet, le code circulaire $X$ identifié dans les gènes est un ensemble de trinucléotides qui a la propriété de retrouver, synchroniser et maintenir la phase de lecture. De plus, il possède la propriété d'autocomplémentarité $\mathcal{C}$ : les trinucléotides de $X$ sont complémentaires entre eux, c'est-à-dire $X=\mathcal{C}(X)$. Enfin, il a la propriété $C^{3}$ : les ensembles de trinucléotides $\mathcal{P}(X)$ et $\mathcal{P}^{2}(X)$ par permutation de $X$ d'un et de deux nucléotides, respectivement, sont également des codes circulaires et de plus complémentaires entre eux, c'est-à-dire $\mathcal{C}\left(X_{1}\right)=X_{2}$ et $\mathcal{C}\left(X_{2}\right)=X_{1}$.

Notre travail de recherche s'est donc intéressé à la recherche de motifs du code circulaire $X$ dans des séquences d'ADN ou d'ARN. Nous avons commencé notre analyse avec le centre de décodage du ribosome (ARNr) qui est une région majeure dans le processus de traduction des gènes aux protéines. Puis, nous avons étendu les résultats obtenus avec le ribosome aux ARN de transfert (ARNt) pour étudier les interactions ARNr-ARNt. Enfin, nous avons généralisé la recherche de motifs de code circulaire $X$ dans l'ADN aux chromosomes d'eucaryotes complets.

La théorie du code circulaire a contribué à l'analyse du centre de décodage du ribosome, en particulier à sa structure primaire. De fa con surprenante, les nucléotides universellement conservés A1492 et A1493 dans tous les ARNr de bactéries, d'archées, d'eucaryotes nucléaires et de chloroplastes appartiennent à des motifs de code circulaire $m_{A A}(X)$. Le nucléotide conservé G530 dans les ARNr des bactéries et des archées est également inclus dans des motifs de code circulaire $m_{G}(X)$. Le développement d'un algorithme de recherche des motifs de code circulaire associé à l'alignement global des séquences multiples permet d'identifier les motifs de code circulaire $m_{G}(X)$ dans les ARNr nucléaires et chloroplastes, résultat qui ne peut pas être obtenu part des méthodes classiques


de bioinformatique. Par ailleurs, un nouveau motif de code circulaire $m(X)$ universellement conservé dans les sept organismes étudiés est identifié gr^ace à la théorie du code circulaire. La visualisation spatiale des trois motifs $m_{A A}(X)$, $m_{G}(X)$ et $m(X)$ montrent qu'ils appartiennent au centre de décodage du ribosome dans tous les ARNr étudiés de bactéries, d'archées, d'eucaryotes nucléaires et de chloroplastes. En conclusion, la fonction biologique du centre de décodage du ribosome qui a été attribuée à un nombre restreint de nucléotides, précisément les nucléotides A1492, A1493 et G530, peut maintenant être associée à des motifs de code circulaire comportant au moins deux et jusqu'à cinq trinucléotides successifs.

Nous identifions également de nouvelles propriétés de cette théorie du code circulaire avec des analyses statistiques de motifs du code circulaire X de grande taille dans les génomes des eucaryotes. Pour la première fois, les régions noncodantes (en dehors des gènes) sont étudiées avec cette théorie du code circulaire. Les motifs du code circulaire $X$ de grande taille de longueurs $l \geq 15$ trinucléotides et de cardinalité (composition) supérieure à 10 trinucléotides ont la plus grande fréquence d'apparition dans les génomes des eucaryotes par rapport (i) aux 23 motifs de codes circulaires bijectifs de grande taille, (ii) aux deux motifs de codes circulaires permutés de grande taille ; (iii) aux motifs aléatoires de grande taille obtenus avec des codes aléatoires (non circulaires). Les plus longs motifs du code circulaire $X$ sont identifiés dans les génomes des eucaryotes, par exemple un motif $X$ dans une région non-codante du génome Solanum pennellii avec une longueur de 155 trinucléotides ( 465 nucleotides) associé à une probabilité d'occurrence de $10^{-71}$, deux motifs $X$ dans des régions non-codante du génome Salmo salar avec des longueurs de 118 trinucléotides ( 354 nucléotides) avec une probabilité d'occurrence de $10^{-52}$, etc. Le plus longs motif du code circulaire $X$ dans le génome humain se trouve dans une région non-codante du chromosome 13 avec une longueur de 36 trinucléotides et une probabilité d'occurrence de $10^{-11}$.

Les motifs du code circulaire $X$ dans les régions non-codantes des génomes sont probablement des vestiges évolutifs de gènes primitifs utilisant le code circulaire pour la traduction des gènes aux protéines. Cependant, les études statistiques réalisées dans ce travail de thèse montrent que les motifs $X$ apparaissent préférentiellement dans les gènes avec une proportion de motifs $X$ (de longueurs l supérieure à 10 trinucléotides et de cardinalité supérieure à 5 trinucléotides) égal à 8 dans les gènes / régions non-codantes pour 138 génomes eucaryotes complets. Ce facteur de 8 est également retrouvé avec les motifs $X$ dans les gènes / régions non-codantes des 24 chromosomes humains. D'un point de vue biologique, cette
propriété peut s'expliquer par le fait que les mutations (substitution, insertion et délétion de nucléotides) sont plus fréquentes dans les régions non-codantes par rapport aux gènes.

L'existence du code circulaire $X$ dans les gènes est un problème ouvert depuis sa découverte en 1996. Nous montrons dans la suite de notre travail que le concept de code circulaire dans les régions d'ADN de faible complexité existe également avec les codes circulaires unitaires ( $U C C$ ) de dinucléotides, trinucléotides et tétranucléotides générant des motifs $U C C$ de dinucléotides répétés ( $D i^{+}$motifs), de trinucléotides répétés (motifs $\mathrm{Tri}^{+}$) et de tétranucléotides répétés (motifs Tetra ${ }^{+}$) dans les génomes d'eucaryotes. Plus précisément, 12 codes $U C C$ de dinucléotides sont "strong comma-free" et quatre d'entre eux $\{A T\},\{C G\},\{G C\}$ et $\{T A\}$ sont auto-complémentaires. Egalement, 48 codes $U C C$ de trinucléotides sont "strong comma-free" et 12 codes $U C C$ de trinucléotides sont "commafree". Enfin, 180 codes $U C C$ de tétranucléotides sont "strong comma-free", 60 codes $U C C$ de tétranucléotides sont "comma-free" et 12 codes "strong commafree" $\{A A T T\},\{A C G T\},\{A G C T\},\{C A T G\},\{C C G G\},\{C T A G\},\{G A T C\}$, $\{G G C C\},\{G T A C\},\{T C G A\},\{T G C A\}$ et $\{T T A A\}$ sont en plus autocomplémentaires. Ainsi, les motifs $\mathrm{Di}^{+}$, $\operatorname{Tr} i^{+}$et Tetra $^{+}$permettent de retrouver une phase de lecture modulo 2 , modulo 3 et modulo 4 , respectivement, dans les régions non-codantes des eucaryotes. De plus, les propriétés $\mathcal{C}^{2}, \mathcal{C}^{3}$ et $\mathcal{C}^{4}$ permettent également de retrouver les phases décalées et la propriété d'autocomplémentarité permet l'appariement des deux brins de l'ADN dans les régions non codantes des eucaryotes. Un motif $U C C$ et son motif $U C C$ complémentaire ont la même distribution dans les génomes des eucaryotes, à la fois selon leur nombre d'apparition et leur nombre total de nucléotides. Cette propriété est observée avec les motifs $D i^{+}, T r i^{+}$et Tetra ${ }^{+}$. De plus, pour les motifs Tri ${ }^{+}$et Tetra ${ }^{+}$, un motif UCC et son motif $U C C$ complémentaire ont des occurrences croissantes inversement proportionnel à leur nombre de liaisons hydrogène.

De manière surprenante, on observe une rareté des trinucléotides répétés (motifs $T r i^{+}$) dans les grands génomes d'eucaryotes par rapport aux motifs $D i^{+}$et Tetra ${ }^{+}$. Ce résultat statistique est obtenu avec des mesures de moyenne et de médiane, et confirmé par deux tests statistiques (un test paramétrique $t$ de Student pour échantillon apparié et un test non-paramétrique $W$ de Wilcoxon signé). Ainsi, les codes circulaires unitaires de trinucléotides associés aux trinucléotides répétés dans les génomes des eucaryotes pourraient avoir contribué à la formation du code circulaire $X$ dans les gènes. Une conséquence d'une telle hypothèse serait la persistance de certaines propriétés statistiques des trinucléotides répétés
du code circulaire X. De fa con inattendue, des paires de trinucléotides identiques (14 trinucléotides parmi 20) du code circulaire $X$ sont préférentiellement utilisées dans les gènes des eucaryotes. Pour la première fois depuis 20 ans, la théorie du code circulaire dans les gènes est étendue aux génomes dans ce travail de thèse. Ainsi, le code circulaire pourrait être une structure mathématique des gènes mais également des génomes.

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Last but not the least, I would like to thank my parents for whom without I would not be here. Therefore, I dedicate this thesis to them.

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## Introduction

We offer here a general introduction of this thesis, realized at The Engineering Science, Computer Science and Imaging (ICube) laboratory. Created in 2013, the laboratory brings together researchers of the University of Strasbourg, the CNRS (French National Center for Scientific Research), the ENGEES and the INSA of Strasbourg in the fields of engineering science and computer science. With around 580 members, ICube is a major driving force for research in Strasbourg.

The work done in this thesis presents a new direction for circular code identified in 1996 by analysing the motifs constructed from circular code. These particular motifs are called circular code motifs. We applied search algorithms to locate circular code motifs in nucleic acid sequences in order to find biological significance. We start with an overview of circular code and its property of coding frame retrieval for genes. Afterwards we present the biological environment in which we applied our work. Finally, we show the structure in which this thesis is presented.

## CIRCULAR CODE

The genetic code consists of 64 trinucleotides $\{A A A, \ldots, T T T\}$, called codons. Each codon encodes for one of the 20 amino acids used in the synthesis of proteins (translation). Most of the amino acids are encoded by more than one codon. Some of the codons have a special purpose. The ATG codon serves as the starting point of translation while also encoding the amino acid methionine at the same time. While the following three trinucleotides $\{T A A, T A G, T G A\}$, called stop codons, are an exception, they do not encode for an amino acid they signal the end of a translation process.

The genetic code can be expressed as either ribonucleic acid (RNA) codons or deoxyribonucleic acid (DNA) codons. RNA codons occur in messenger RNA (mRNA) and are the codons that are actually read during translation. Each mRNA molecule acquires its sequence of nucleotides by transcription from the corresponding gene. Genes are DNA sequences which are read modulo 3 letters among the three possible frames. As such, only one frame, called reading
frame, which begins with a start codon and ends with a stop codon, codes the corresponding protein sequence according to the genetic code.

However, this does not mean a translation starts at every $A T G$ codon, even though it accounts for most start codons, it could be one of the following $\{T T G, G T G, C T G\}$. Additional requirements need to be present when assigning a start codon, such as the ribosome biding side, the shine-delgarno sequence. This short sequence needs to be located 7 to 13 bases upstream of the start codon. Add to that the importance of maintaining the correct reading frame.

All this indicates that the procedure of maintaining the correct reading frame of genes is far more complex. It was theorized that there are sets of trinucleotides called circular codes $X$ which have the property of reading frame retrieval, synchronization and maintenance (Michel, 2012). Furthermore, there are circular codes which have in addition the $C$ self-complementary property, i.e. the trinucleotides of $X$ are complementary to each other, i.e. $X=\mathcal{C}(X)$. Finally, there are self-complementary circular codes $X$ which have in addition the $C^{3}$ property, i.e. the permuted trinucleotide sets $\mathcal{P}(X)$ and $\mathcal{P}^{2}(X)$ of $X$ by one and two nucleotides, respectively, are also trinucleotide circular codes and complementary to each other, i.e. $\mathcal{C}\left(X_{1}\right)=X_{2}$ and $\mathcal{C}\left(X_{2}\right)=X_{1}$. In 1996, a $C^{3}$ self-complementary trinucleotide circular code $X$ has been identified in genes (reading frame of mRNAs) simultaneously in eukaryotes and prokaryotes (Arquès and Michel, 1996).

## Biological context

Our work revolves around searching for $X$ circular code motifs in DNA or RNA sequences. A circular code motif is basically a sequence of nucleotides where its trinucleotides belong to the circular code. In our biological environment we approached the matter by addressing the very specific and narrow, then moving to the more general and broader look. The first part of the study was confined to the ribosome, more specifically, we started with the decoding center of the ribosome. A region that is very important to the translation process. Afterwards, we included the transfer RNA in the ribosome in the study, in order to give a wider look at the circular code motifs in the ribosome and whether a possible interaction exist. Later in our work, to address the presence of $X$ circular code motifs in DNA, we searched the entire database of complete eukaryotic chromosomes. This involved a huge amount of data and computation. Our results from circular code were thoroughly compared to those of random generated codes in
order to establish the significance of our findings.

## Thesis structure

This thesis is structured into an introduction, four chapters for the main body and a conclusion.

The first chapter will present the biological environment we are working in. Our main focus lies in the nucleic acid sequence, we will be explaining what is a sequence made of. We worked in two different biological contexts, the first was the ribosome, in order to highlight the importance of our findings we explaining briefly the translation process and shed some light on the 3D structure of the ribosome. The second phase of our work involved the sequence of complete eukaryotic chromosomes, which was acquired from the RefSeq database.

The second chapter serves as an introduction of circular code, beginning with the root that lead to its discovery. This takes us back to the race of cracking the genetic code, as many code theories fell in light of reality once the genetic code was finally cracked. We will explain how the circular code came to be after the discovery of the genetic code, its features and significance. Finally, we present what are circular code motifs, which serve as the main study course of this thesis.

The data and methods are presented in the fourth chapter. We present the data obtained from the databases mentioned before. A new algorithm was written to extract motifs from sequences, this algorithm is versatile as it can take any code as a parameter. Multiple sequence alignments were modified to allow us to switch the attention onto circular code motifs, this combination allowed us to find interesting results in the ribosome. The chapter also explains how the huge data from the complete eukaryotic chromosomes was approached. We show the various codes we used and the statistical methods employed to compare them.

Our results are presented, detailed and discussed in the fourth chapter. The amount of data that we extracted was just huge. This extensive chapter shows the discoveries found in the decoding center of the ribosome, while also raising some questions about several interesting motifs found in the ribosome and offer an intriguing take on the structure of the tRNA. We also present an deep statistical analysis of the significance and uniqueness of the $X$ circular code, discovered in 1996 (Arquès and Michel, 1996), when compared to other codes, whether they are circular, bijective transformation or randomly generated.

Finally, we give the conclusions from our various findings while providing
some theories, as our work raises questions on the importance and role of the circular code while highlighting new properties.

## Biological environment

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### 1.1 Introduction

In this chapter we will define the biological environment in which we are working.
This thesis can be divided into two different sub studies with respect to the biological environment. In the first half of this study we searched for circular code motifs in ribosomes, the cellular protein factory. The study focused on the presence of circular code motifs in important areas of the ribosome. To accomplish this the data used includes the 3D structure of the ribosome and a spatial examination of these motifs. The second part of our work involves a more general study and a wider scope. The sequences of complete eukaryotic chromosomes were retrieved and searched for circular code motifs.

We will give brief description of the nucleic acid sequence, which constitute our base target of research, and then introduce the ribosome while explaining its function during translation. Finally, we explain the nature of the data we are using and its source.

### 1.2 NUCLEIC ACID SEQUENCE

Nucleosides that have one or more phosphate groups attached to the sugar are called nucleotides, those containing ribose $(\mathrm{OH})$ are known as ribonucleotides while those containing deoxyribose (H) are known as deoxyribonucleotides (Figure 1.1). Nucleobases are nitrogen-containing rings linked to a sugar within a nucleoside, historically called simply as bases. These bases are grouped into two families depending on strong resemblance, Cytosine $(C)$, Thymine $(T)$, and Uracil $(U)$ are called pyrimidines while guanine $(G)$ and adenine $(A)$ are purines. Each nucleotide is named after the base it contains.


Pyrimidines




Figure 1.1: Structural elements of a nucleotide: nucleoside in green, bases in blue (with their groups) and the different phosphate groups in red. (By Boris [Public domain], via Wikimedia Commons).

Nucleotides are linked together by the formation of covalent bonds between the phosphate group attached to the sugar of a nucleotide and the hydroxyl group on the sugar of the next nucleotide (Figure 1.2). These links will form a nucleic acid polymer. Nucleotides are therefore responsible for the storage and retrieval of biological information.

Nucleic acids differ in the type of sugar contained in their sugar backbone. Those based on the sugar ribose are known as ribonucleic acids (RNA), and contain the bases $A, G, C$, and $U$. They are generally a single-stranded polynucleotide chain. While based on deoxyribose are known as deoxyribonucleic acids (DNA), and contain the bases $A, G, C$, and $T$ ( $T$ is chemically similar to the $U$ in RNA). They are a double helix composed of two polynucleotide chains that run in opposite directions and are held together by hydrogen bonds between the bases of the two chains (Figure 1.3).


Figure 1.2: An example of phosphodiester bonds $\left(\mathrm{PO}_{4}^{3-}\right)$ between Thymine ( T ) and two molecules of Adenine (A). (By G3pro [Public domain], from Wikimedia Commons).


Figure 1.3: Chemical structure of DNA. The two chains run in opposite directions. (By Madeleine Price Ball [Public domain], via Wikimedia Commons).

These two types of nucleic acid hold the genetic information in all life forms, but they differ in roles. On one hand the DNA is more stable because of the double helix structure making it ideal for long term keeping, on the other hand
the RNA is a transient carrier of information.

### 1.3 Ribosome

The ribosome, a cellular organelle, is a complex macromolecule consisting of RNAs and proteins. It is responsible for the synthesis of the cell protein by translating specific genetic information that is encoded in the deoxyribonucleic acid (DNA) of the cell genome and transferred to the ribosome by messenger RNA (mRNA).

A ribosome is composed of two subunits, a large subunit and a small subunit. Each subunit is an assembly of ribosomal RNAs (rRNAs) and ribosomal proteins. The small subunit is responsible for initiation, identification of the correct reading frame and encoding of the genetic code. The main chemical reaction of protein synthesis, peptide bond formation, occurs in the large subunit. A ribosome contains three transfer RNA (tRNA) binding sites: A-site (aminoacyl), P-site (peptidyl), and E-site (exit).

### 1.3.1 Translation

Translation is the process of adding one amino acid after the other to the growing polypeptide chain until the protein synthesis in done. Initially the two before mentioned subunits of the ribosome come together on an mRNA at it's 5 ' end.

At first, the aminoacyl tRNA carrying the amino acid binds to the A-site where the decoding center containing the universally conserved nucleotides G530, A1492 and A1493 of the smaller rRNA subunit is tasked with distinguishing cognate from non-cognate tRNAs by anticodon-codon interactions with the mRNA codon (Wilson, 2014). After the aminoacyl-tRNA binds to the corresponding codon on the mRNA, a peptide-bond forms between the carboxyl end of the polypeptide chain at the P-site and the new arrived amino-acid at the A-site. This reaction is catalysed by an enzymatic site in the large subunit. Consequently, the larger subunit shifts relatively to the smaller subunit, thus moving the tRNAs in the A and P sites to the P and E sites respectively. Following this, the small subunit will move exactly three nucleotides along the mRNA, aligning itself with the large subunit. This will incur a reset, where the tRNA at the E-site is ejected while the A-site is empty for a new tRNA. Given that the mRNA is being translated in the direction of the 3 ' from the 5 ', means that the protein is formed first from its N -terminal end all the way to its C -terminus.


Figure 1.4: The mechanics of ribosome during translation. The start of the aminoacyl tRNA at the $A$-site, then the transition to the P -site and the exit through the E -site. Showing the position of mRNA with respect to the smaller ribosomal subunit and the various tRNAs. (By LadyofHats [Public domain], via Wikimedia Commons).

When synthesis of the protein is finished, the two subunits of the ribosome separate. Translation speed varies between domains, between six and nine amino acids per seconds for eukaroytes, while it is between seventeen and twenty-one for prokaryotes (Reuveni et al., 2011).

### 1.3.2 Biomolecular structure

Understanding the functionality of the ribosome was accomplished by determining its three-dimensional structure. While several methods exist to determine the structure of a protein, two are most commonly used, with a third method steadily on the rise (Figure 1.8).

The first of which is X-ray crystallography, where the protein is purified and crystallized. Given the tiny wavelength of X-rays ( 0.1 nanometre), scientists can then probe the structure of very small objects at the atomic level. Interpreting the resulting map of electron density determines the location of each atom. The second method is Nuclear Magnetic Resonance (NMR) spectroscopy, where a protein is purified, placed in a strong magnetic field then subjugated to a blast of radio waves. These steps will align the atoms according to the magnetic field,


Figure 1.5: (a), (b), The overall conformations of universally conserved G530, A1492 and A1493 of 16S rRNA in the cognate structures are identical to those in the near-cognate models when the mismatches are at the first (a) or second (b) codon-anticodon positions. (c), differences between the position of the first uridine in the $U U U$ codon base-paired to the $G A G$ anticodon of $\mathrm{tRNA}_{2}{ }^{\text {Leu }}$ from the 70 S structure and from the 30 S model. (d), Comparison of the anticodon loops of tRNA ${ }^{\mathrm{Tyr}}$ in the cognate (red) and near-cognate (cyan) states. (e), Rearrangements of rRNA helices h44 and H69 in the near-cognate state upon binding of the aminoglycoside paromomycin (PAR). The near-cognate structures with RRNA $^{\text {Tyr }}$ are shown. (f), Magnified view of the changes in the A1493 phosphate position (Demeshkina et al., 2012).
which the radio waves will then disturb briefly before returning to their aligned position. This will allow scientist to study the relative position of these atoms in a protein. The third method is Electron Microscopy (EM), where a beam of electrons is used to image the molecule directly. Typically, EM experiments are combined with information from X-ray crystallography or NMR spectroscopy for atomic details mainly due for present limitation of EM.


Figure 1.6: Crystallographic structure of the smaller subunit of Escherichia coli ribosome. The 3D positioning of mRNA in green, the three tRNAs in different shades of blue and the smaller ribosomal subunit (Generated using Jmol).

Determining the structure of a protein doesn't rely on these methods only, in most cases prior knowledge is necessary. Such as amino acid sequence and preferred geometry of atoms.

### 1.4 Biological databases

We will be presenting now the different biological databases used to retrieve the data on which the studies were conducted. For the first part of the study we used crystallographic data to examine if what we are searching for is in significant regions of the ribosome. In the second part of the study, we used large quantity of data of genomes to thoroughly examine the circular code in eukaryotic organisms.


Jmol
Figure 1.7: Crystallographic structure of the Escherichia coli ribosome. The 3D positioning of mRNA in green, the three tRNAs in different shades of blue, the smaller ribosomal subunit in light brown and the large subunit in orange (Generated using Jmol).

### 1.4.1 Crystallographic database

Crystallographic databases are created with the goal of collecting information about the structure of molecules and crystals. The protein data bank (PDB) archive is such a database (www.rcsb.org). Structural biologists use methods such as X-ray crystallography, NMR spectroscopy, and cryo-electron microscopy to construct a 3D structure of proteins. This information is deposited in the PDB , which is then annotated and publicly released into the archive by the worldwide protein data bank (wwPDB), an organization that is responsible with maintaining the archive.

PDB holds structures for proteins and nucleic acids, such as ribosomes, oncogenes, drug targets and complete viruses. Multiple structures can exist for the same molecule depending on the test conducted or the scope of the study. The files found in the database consist in principle of, the atoms in each protein and their three-dimensional coordinates, a header that summarizes the input and the experiments in which this structure was acquired.


Figure 1.8: The increase of crystallographic structure submissions to the PDB according to methods. X-ray crystallography (blue) and Nuclear Magnetic Resonance (green) were the dominant methods while Electron Microscopy (red) is becoming more popular with the years (numbers provided by PDB).

Molecular graphics software are available to visualize these files in 3D, such as Jmol, RasMol, Swiss PDB viewer, ...etc, with additional features such as measuring distances and bond angles. These software allow us to carefully study and structure and identify interesting structural features.

### 1.4.2 SEQuence database

The National Center for Biotechnology Information (NCBI) boasts an array of biological database (www.ncbi.nlm.nih.gov), bioinformatic tools and services. Particularly we are interested in Reference Sequence (RefSeq) and Genbank databases.

RefSeq (www.ncbi.nlm.nih.gov/refseq) is a large multi-species, nonredundant, curated sequence database consisting everything from transcripts and translation products to whole genomes. While the database is non-redundant, alternate assemblies of the same sequence can exist. RefSeq employs a strict curative process where a record may be an essentially unchanged, validated copy of the original submission, or include updated or additional information supplied
by collaborators or NCBI staff.
GenBank (www.ncbi.nlm.nih.gov/genbank), on the other hand, a redundant archival database is an annotated collection of all publicly available nucleotide sequences and their protein translations submitted directly by individual laboratories, as well as from bulk submissions from large-scale sequencing centres. GenBank continues to grow at an exponential rate, doubling every 10 months.

### 1.5 Summary

In this chapter we presented the nucleic acid sequences (DNA and RNA) and their composition. Then, we preceded to explain the translation process inside the ribosome and how a protein is synthesised inside a cell. Finally, we shed light on the bimolecular structure of a ribosome. This biological introduction is to help us better understand how we study circular code in a nucleic acid sequence. The 3D structure of a ribosome is vital for our first part of the study, where we focus on the decoding center of the ribosome and then move on to enlarge the scope of the study to examine the area around the decoding center and possible interactions with the tRNAs present in a ribosome at the time of translation.

We mentioned as well the nature of data we are working with. Crystallographic structures were an essential part when it came to figure out the working mechanics of a ribosome. As such, crystallographic databases were vital for us to understand and study circular code motifs presence in a ribosome. Finally, the sequence database RefSeq that houses a huge amount of complete chromosomes proved excellent for us to conduct a study on the entire set of eukaryotic genomes published at the time.

In the next chapter, we will start with a brief history of codes, and how the discovery of DNA structure ushered the race to crack the genetic code. Afterwards, we will present the circular code, how it was discovered and why it is an interesting study due to the many properties it has.

## 2

## Code Theory

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### 2.1 Introduction

In this chapter we will be explaining what is circular code, how it came to be, and in order to do that we will begin with a brief history of how code theory began and its purpose. We will mention codes that were very important at the time of their inception and why they were dismissed eventually. Finally, we explain what are circular code motifs while shedding light a subclass called unitary circular codes that are interesting for us.

### 2.2 History of codes

The discovery of the double-helix structure of DNA (Watson and Crick, 1953) raised the question of how to translate a 4 letter alphabet into 20 words? The first deduction was it cannot be a one-to-one mapping, and a two letter word results in 16 words, still short of 20 . Therefore, the representation could not be smaller than a three letter word (trinucleotide). But that would also give 64 words, which is an excess to the 20 amino acids. Scientist shortly rushed to crack the secret of genetic expression. This lead to the publication of several codes hoping to answer this question.

### 2.2.1 DiAmond code

The first to propose a coding scheme following the Watson-Crick structure was George Gamow, better known for his work on the Big Bang theory.

Called the diamond code (Gamow, 1954), it suggested that double-stranded DNA acted directly as a template for assembling amino acids into proteins. Gamow envisioned the grooves in the double helix as holes that would fit the side chains of amino acids in a "key-and-lock" fashion (Figure 2.1). Even though the diamond have four corners, only three of them are utilized because the paired bases on the horizontal diagonal are complementary. This is essence makes the diamond code a triplet code. Figure 2.1 show that there is 20 distinct holes.

Gamow reasoned that 12 holes are symmetrical, these diamonds could be flipped end-for-end or flopped side-to-side without changing their meaning. This allowed the possibility of several triplets coding for the same amino acid, thus dealing with the problem of having 64 trinucleotides and 20 amino acids.

This code had another feature that lead to its eventual dismissal. Each nucleotide was simultaneously present in three trinucleotides, for example, a
base sequence $A C G T A A$ would result in four overlapping trinucleotides: $A C G$, $C G T, G T A, G T A$. Which was proven to be wrong.


Figure 2.1: George Gamow's diamond code used the key-and-lock between the grooves found in DNA and the amino acids (Gamow, 1954).

### 2.2.2 Comma-free code

Proposed by Crick, Griffith, and Orgel, 1957, comma-free code was an effort to study the encoding of three nucleotides $\{A A A, A A C, \ldots, T T T\}$ into amino acids. By excluding the four periodic permuted trinucleotides $\{A A A, C C C, G G G, T T T\}$ and distributing the 60 remaining trinucleotides into 20 groups of three trinucleotides such that each group contains the set of trinucleotides that can be permuted from each other following the circular permutation map (Definition 2.1). Based on this we can deduce that a comma-free code can have only one trinucleotide from each group, therefore a set contains at most 20 trinucleotides.

Notation 2.1. The nucleotides define the genetic alphabet $B=\{A, C, G, T\}$. The set of non-empty words (words, respectively) over $B$ is denoted by $B^{+}\left(B^{*}\right.$, respectively). The set of the 64 words of length 3 (trinucleotides or triletters) on $B$ is denoted by $B^{3}=\{A A A, \ldots, T T T\}$. Let $x_{1} \cdots x_{n}$ be the concatenation of the words $x_{i}$ for $i=1, \ldots, n$, the symbol " . " being the concatenation operator.

Notation 2.2. In genes, there are three frames $f$. By convention here, the reading frame $f=0$ is established by a start codon $\{A T G, G T G, T G T, T T G\}$ and the frames $f=1$ and $f=2$ are the reading frame $f=0$ shifted by one and two nucleotides in the $5^{\prime}$-to- 3 ' (left to right) direction, respectively.

Definition 2.1. The trinucleotide circular permutation map $\mathcal{P}: B \rightarrow B$ is defined by $\mathcal{P}\left(l_{0} \cdot l_{1} \cdot l_{2}\right)=l_{1} \cdot l_{2} \cdot l_{0}$ for all $l_{0}, l_{1}, l_{2} \in B$, e.g. $\mathcal{P}(A T G)=T G A$. The second iterate of $\mathcal{P}$ is denoted as $\mathcal{P}^{2}$, e.g. $\mathcal{P}^{2}(A T G)=G A T$. By extension to a trinucleotide set $S$, the set circular permutation map $\mathcal{P}: \mathbb{P}\left(B^{3}\right) \rightarrow \mathbb{P}\left(B^{3}\right)$, $\mathbb{P}$ being the set of all subsets of $B^{3}$, is defined by $\mathcal{P}(S)=\left\{v \mid u, v \in B^{3}, u \in\right.$ $S, v=\mathcal{P}(u)\}$, i.e. a permuted trinucleotide set $\mathcal{P}(S)$ is obtained by applying the circular permutation map $\mathcal{P}$ to all its trinucleotides, e.g. $\mathcal{P}(\{A C G, A G T\})=$ $\{C G A, G T A\}$ and $\mathcal{P}^{2}(\{A C G, A G T\})=\{G A C, T A G\}$.

Despite having an identical number of trinucleotides as amino acids, no trinucleotide comma-free code was identified in genes. Early in the sixties it was discovered that the trinucleotides $T T T$, an excluded trinucleotide in comma-free code, in fact codes phenylalanine (Nirenberg and Matthaei, 1961), this in turn would lead to the abandonment of comma-free code.

Definition 2.2. A set $S \subset B^{+}$of words is a code if, for each $x_{1}, \ldots, x_{n}$, $y_{1}, \ldots, y_{m} \in S, n, m \geq 1$, the condition $x_{1} \cdots x_{n}=y_{1} \cdots y_{m}$ implies $n=m$ and $x_{i}=y_{i}$ for $i=1, \ldots, n$, e.g. $B^{3}=\{A A A, \ldots, T T T\}$ is a code, where as $X=\{A, G C, A G C\}$ is not a code as there are two decompositions $A \cdot G C=A G C$.

Definition 2.3. (Fimmel, Michel, and Strüngmann, 2016) Let $X \subseteq B^{m}, m \in \mathbb{N}$ with $m \geq 2$, be an $m$-nucleotide code. The directed graph $\mathcal{G}(X)=(V(X), E(X))$ associated with $X$ has a set of vertices $V(X)$ and a set of edges $E(X)$ defined as follows:

$$
\left\{\begin{array}{l}
V(X)=\left\{N_{1} \ldots N_{i}, N_{i+1} \ldots N_{m}: N_{1} \ldots N_{m} \in X, 1 \leq i \leq m-1\right\}  \tag{2.1}\\
E(X)=\left\{\left[N_{1} \ldots N_{i}, N_{i+1} \ldots N_{m}\right]: N_{1} \ldots N_{m} \in X, 1 \leq i \leq m-1\right\}
\end{array}\right.
$$

Theorem 2.1. (Fimmel, Michel, and Strüngmann, 2016) Given an m-nucleotide code $X \subseteq B^{m}, m \geq 2$, the following statements are equivalent:

1. The code $X$ is comma-free.
2. The maximal length of a path in $\mathcal{G}(X)$ is 2 .

Theorem 2.2. (Fimmel, Michel, and Strüngmann, 2016) Given an $m$-nucleotide code $X \subseteq B^{m}, m \geq 2$, the code $X$ is strong comma-free if the maximal length of a path in $\mathcal{G}(X)$ is 1 .

### 2.2.3 Genetic code

In 1961, Marshall Nirenberg and Heinrich Matthaei managed to crack the first word of the genetic code. They performed an experiment which showed that a chain of the repeating bases $U$ (Uracil) forced a protein chain made of one repeating amino acid, phenylalanine. This was a breakthrough experiment which proved that the code could be broken.

After the initial discovery the team grew in size to replicate the poly- $U$ experiment model to other amino acids. Using 20 test tubes for each amino acid, respectively, they experimented with the different 64 combination of nucleotides to form three letter words.

By 1966 the genetic code was complete, all the mappings between the 64 codons and the 20 amino acids were established. As it turns out, there was no pattern in the code. Some amino acids were represented by one or two codons, some by more, ignoring all mathematical approaches to solving this coding mystery.

### 2.3 Circular code

In 1996, a statistical analysis of occurrence frequencies of the 64 trinucleotides was conducted in the three frames (Definition 2.2), of genes of both prokaryotes and eukaryotes. The study showed that the trinucleotides are not uniformly distributed in the three frames (Arquès and Michel, 1996). By convention here, the frame zero is the reading frame in a gene, and the frames 1 and 2 are the reading frame 0 shifted by 1 and 2 nucleotides in the $5^{\prime}$-to- 3 ' direction, respectively. By excluding the four periodic permuted trinucleotides $\{A A A, C C C, G G G, T T T\}$ and by assigning each trinucleotide to a preferential frame (frame of its highest occurrence frequency), three subsets $X=X_{0}, X_{1}$, and $X_{2}$ of 20 trinucleotides each, were assigned to frame 0,1 and 2 , respectively (Table 2.1). The analysis was based on the large gene populations (protein coding regions) of eukaryotes ( 26,757 sequences, $11,397,678$ trinucleotides) and prokaryotes ( 13,686 sequences, 4,709,758 trinucleotides) (Arquès and Michel, 1996). The following circular code $X$ was observed in frame 0 (reading frame):

Table 2.1: Frequency of trinucleotides occurrences according to frame 0,1 and 2 in genes sequences from eukaryotes and prokaryotes (Arquès and Michel, 1996). The table shows the occurrences of the first seven trinucleotides from $B^{3}$, with their preferred frame in bold.

|  | Frequency(\%) |  |  |
| :--- | ---: | ---: | ---: |
| Trinucleotide | Frame 0 | Frame 1 | Frame 2 |
| AAA | $\mathbf{3 . 3 8}$ | 2.75 | 2.44 |
| AAC | $\mathbf{2 . 1 8}$ | 1.59 | 1.38 |
| AAG | 1.98 | $\mathbf{3 . 2 1}$ | 0.81 |
| AAT | $\mathbf{2 . 1 7}$ | 1.37 | 1.69 |
| ACA | 1.22 | $\mathbf{1 . 9 1}$ | 1.11 |
| ACC | $\mathbf{2 . 0 9}$ | 1.60 | 0.79 |
| ACG | 1.30 | $\mathbf{2 . 4 9}$ | 0.68 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |

$$
\begin{align*}
X=\{ & A A C, A A T, A C C, A T C, A T T, C A G, C T C, C T G, G A A, G A C,  \tag{2.2}\\
& G A G, G A T, G C C, G G C, G G T, G T A, G T C, G T T, T A C, T T C\} .
\end{align*}
$$

The two sets $X_{1}$ and $X_{2}$, of 20 trinucleotides each, in the shifted frames 1 and 2 of genes can be deduced from $X$ by the circular permutation map (Definition 2.1), i.e. $X_{1}=\mathcal{P}(X)$ and $X_{2}=\mathcal{P}^{2}(X)$.

Definition 2.4. The nucleotide complementarity map $\mathcal{C}: B \rightarrow B$ is defined by $\mathcal{C}(A)=T, \mathcal{C}(C)=G, \mathcal{C}(T)=A$ and $\mathcal{C}(G)=C$. According to the property of the complementary and anti-parallel double helix, the trinucleotide complementary map $\mathcal{C}: B^{3} \rightarrow B_{4}^{3}$ is defined by $\mathcal{C}\left(l_{0} \cdot l_{1} \cdot l_{2}\right)=\mathcal{C}\left(l_{2}\right) \cdot \mathcal{C}\left(l_{1}\right) \cdot \mathcal{C}\left(l_{0}\right)$ for all $l_{0}, l_{1}, l_{2} \in B$, e.g. $\mathcal{C}(A T G)=C A T$. By extension to a trinucleotide set $S$, the set complementarity map $\mathcal{C}: \mathbb{P}\left(B^{3}\right) \rightarrow \mathbb{P}\left(B^{3}\right)$, is defined by $\mathcal{C}(S)=\left\{v \mid u, v \in B^{3}, u \in S, v=\mathcal{C}(u)\right\}$, i.e. a complementary trinucleotide set $\mathcal{C}(S)$ is obtained by applying the complementarity map $\mathcal{C}$ to all its trinucleotides, e.g. $\mathcal{C}(\{A C G, A G T\})=\{A C T, C G T\}$.

Definition 2.5. A trinucleotide code $X \subset B^{3}$ is circular code if, for each $x_{1}, \ldots, x_{n}, y_{1}, \ldots, y_{m} \in X, n, m \geq 1, r \in B^{*}, s \in B^{+}$, the conditions $s x_{2} \cdots x_{n} r=$ $y_{1} \cdots y_{m}$ and $x_{1}=r s$ imply $n=m, r=\varepsilon$ (empty word) and $x_{i}=y_{i}$ for $i=1, \ldots, n$ (Figure 2.2 for a graphical representation of the circular code definition).

Theorem 2.3. (Fimmel, Michel, and Strüngmann, 2016) Given an $m$-nucleotide code $X \subseteq B^{m}, m \in \mathbb{N}$ with $m \geq 2$, the following statements are equivalent:

1. The code $X$ is circular.
2. The graph $\mathcal{G}(X)$ is acyclic.

Definition 2.6. An $m$-nucleotide unitary circular code $X \subseteq B^{m}$ (UCC) is a code with a unique word.


Figure 2.2: Graphical representation of the circular code definition (Arquès and Michel, 1996).


Figure 2.3: The following code $\{A A T, A T G, C C T, C T A, G C C, G G C\}$ is not circular, since ( $A T G, G C C, C T A$ ) can be read differently if we shift frames showing us ( $A A T, G G C, C C T$ ) (Michel, 2012).

Remark 2.1. A trinucleotide code $C$ containing either one periodic permuted trinucleotide $P P T=\{A A A, C C C, G G G, T T T\}$ or two non-periodic permuted trinucleotides $N P P T=\{t, \mathcal{P}(t)\}$ for a trinucleotide $t \in B^{3} \backslash P P T$ cannot be circular. Thus, the two trinucleotide codes $B^{3}$ and $B^{3} \backslash P P T$ are not circular.

Remark 2.2. The fundamental property of a circular code is the ability to retrieve the reading (original or construction) frame of any sequence generated with this circular code. A circular code is a set of words over an alphabet such that any sequence written on a circle (the next letter after the last letter of the sequence being the first letter) has a unique decomposition (factorization) into words of the circular code (Michel, 2012) (Figure 2.3 for an example). The reading frame in a sequence (gene) is retrieved after the reading of a certain number of letters (nucleotides), called the window of the circular code. The length of this window
for retrieving the reading frame is the letter length of the longest ambiguous word, not necessarily unique, which can be read in at least two frames, plus one letter (Figure 2.4).


Figure 2.4: An example of how a window can determine in which frame the circular code motif is, if size is sufficient (13 nucleotides). Retrieval of the reading frame of the word $w=\ldots$ AGGT AATT ACCAG $\ldots$ of the trinucleotide circular code $X$ (Equation 2.2). Among the three possible factorizations $\tilde{w}_{0}, \tilde{w}_{1}$ and $\tilde{w}_{2}$, only one factorization $\tilde{w}_{1}$ into trinucleotides of $X$ is possible leading to $\ldots A \cdot G G T \cdot A A T \cdot T A C \cdot C A G \cdot \ldots$ Thus, the first letter $A$ of $w$ is the 3rd letter of a trinucleotide of $X$ (Michel, 2012).

Remark 2.3. At window length $l>12$ nucleotides there is no ambiguous word of the circular code $X$.

Definition 2.7. A trinucleotide set $X_{1}=\mathcal{P}(X)$ of a trinucleotide circular code $X$ is permuted if, for each $x \in X, \mathcal{P}(x) \in \mathcal{P}(X)$. The permuted trinucleotide set $X_{2}=\mathcal{P}^{2}(X)$ is defined similarly.

Definition 2.8. A trinucleotide circular code $X \subset B^{3}$ is maximal if for each $x \in B^{3}, x \notin X, X \cup\{x\}$ is not a trinucleotide circular code.

Definition 2.9. A trinucleotide circular code $X$ is self-complementary if, for each $x \in X, \mathcal{C}(x) \in X$.

Definition 2.10. An $m$-nucleotide circular code $X \subseteq B^{m}$ is $C^{n}$ if the $m$ permuted m-nucleotide codes $X_{1}=\mathcal{P}(X), \ldots, X_{m}=\mathcal{P}^{m}(X)$ are circular. An $m$-nucleotide comma-free $X \subseteq B^{m}$ (strong comma-free, respectively) is $C F^{m}\left(S C F^{m}\right.$, respectively) if the $m$ permuted $m$-nucleotide codes $X_{1}=$ $\mathcal{P}(X), \ldots, X_{m}=\mathcal{P}^{m}(X)$ are comma-free (strong comma-free, respectively). A trinucleotide circular code $X \subset B^{3}$ is $C^{3}(\mathrm{~m}=3)$ if the permuted trinucleotide sets $X_{1}=\mathcal{P}(X)$ and $X_{2}=\mathcal{P}^{2}(X)$ are trinucleotide circular codes.

Definition 2.11. A trinucleotide circular code $X \subset B^{3}$ is $C^{3}$ self-complementary if $X, X_{1}=\mathcal{P}(X)$ and $X_{2}=\mathcal{P}^{2}(X)$ are trinucleotide circular codes satisfying the following properties $X=\mathcal{C}(X), \mathcal{C}\left(X_{1}\right)=X_{2}$ and $\mathcal{C}\left(X_{2}\right)=X_{1}$.

Table 2.2: The number of sets that can be circular for each class of trinucleotide (Arquès and Michel, 1996; Michel and Pirillo, 2010; Michel, Pirillo, and Pirillo, 2012).

| Circular code (maximal) | Number |
| :--- | ---: |
| Potential | 3486784401 |
| Identified | 12964440 |
| Self-complementary | 528 |
| $C^{3}$ | 221328 |
| $C^{3}$ Self-complementary | 216 |

A circular code containing 20 codons is designated as maximal (Definition 2.8). The number possible maximal circular codes is $3^{20}$ out of $64^{20}$, i.e. probability of $10^{-27}$, only 12964440 are effectively circular codes. Which makes finding these three circular code sets in genes quite intriguing. These three trinucleotide sets posses several strong mathematical properties. They have the fundamental property to always retrieve the reading frame in any position of any sequence generated with the circular code. Initiation and stop trinucleotides, as well any frame signals are not necessary to define the reading frame. A window of 13 nucleotides allows to retrieve the reading frame for all the ambiguous words generated with $X$. Therefore, circular codes are less constrained than the comma-free codes. Moreover, the $X_{0}$ code is in particular very interesting since it is symmetric under the complementary transformation. In other words, we exchange every trinucleotide in the set with its complement (Definition 2.4), the set remains unchanged. While the $X_{1}$ and $X_{2}$ sets are complementary to each other.

### 2.3.1 Circular code motifs

A circular code motif is a phrase composed by words, in this case trinucleotides, from a circular code. Consequently, an $X$ motif can be found in any DNA or RNA sequence, where we have successive trinucleotides from the $X$. As mentioned above, a window of 13 nucleotides is sufficient to distinguish a circular code motif in any given frame. Let us examine the following example:

## Example 2.1.

Sequence: AGTC AGT AGCTGAGGCAGCTCGAAATTCGT
Comma separated: $A G T, C A G, T A G, C T G, A G G, C A G, C T C, G A A, A T T, C G T$

Consider the DNA sequence in example 2.1. If we read this sequence in frame 0, we can see the four following consecutive trinucleotides $\{C A G, C T C, G A A, A T T\}$ belonging to $X$ (Equation 2.2). Therefore, CAGCTCGAAATT is an $X$ motif that starts at nucleotide 16 and ends at nucleotide 27 according to the sequence.

Definition 2.12. An $m$-nucleotide unitary circular code motif (UCC motif) generated by an $m$-nucleotide unitary circular code (Definition 2.6), is a concatenation of $n$ words $w$ of size $m$ denoted $w^{n}$. The class of the motifs $w^{n}$ for all $n$ is denoted by $m^{+}$.

Definition 2.13. Two $U C C$ motifs $w_{1}^{+}$and $w_{2}^{+}$are said equivalent if $w_{1}^{+}$and $w_{2}^{+}$ are related by the circular permutation map $\mathcal{P}$ (Definition 2.1).

### 2.3.2 Unitary circular codes of dinucleotides, trinucleotides and TETRANUCLEOTIDES

The $4^{2}-4=12$ dinucleotide unitary codes $D i=\left\{l_{1} l_{2}\right\}$ with $l_{1} l_{2} \in B$ and $l_{1} \neq l_{2}$ are circular and $C^{2}$. The $4^{3}-4=60$ trinucleotide unitary codes Tri $=\left\{l_{1} l_{2} l_{3}\right\}$ with $l_{1}, l_{2}, l_{3} \in B$ and $l_{1} l_{2} \neq l_{2} l_{3}$ are also circular and $C^{3}$. The $4^{4}-16=240$ tetranucleotide unitary codes Tetra $=\left\{l_{1} l_{2} l_{3} l_{4}\right\}$ with $l_{1}, l_{2}, l_{3}, l_{4} \in B$ and $l_{1} l_{2} \neq l_{3} l_{4}$ are also circular and $C^{4}$ (excluding $\left\{l_{1} l_{2} l_{1} l_{2}\right\}$ since it is not circular, $\left(l_{1} l_{2}\right)^{2}$ is a dinucleotide repeat of $\left.\left\{l_{1} l_{2}\right\}\right)$. We describe some additional stronger combinatorial properties for the codes $D i, T r i$ and Tetra. From Theorem 2.2, an $m$-nucleotide unitary code $\left\{l_{i} \ldots l_{i}\right\}$ with $l_{i} \in B$, i.e. starting and ending by the same letter, cannot be strong comma-free.

### 2.3.2.1 UNITARY CIRCULAR CODES OF DINUCLEOTIDES

The 12 dinucleotide unitary codes $\left\{l_{1} l_{2}\right\}$ and $\left\{P\left(l_{1} l_{2}\right)\right\}=\left\{l_{2} l_{1}\right\}$ with $l_{1}, l_{2} \in B$ and $l_{1} \neq l_{2}$, i.e. $\{A C\},\{A G\},\{A T\},\{C A\},\{C G\},\{C T\},\{G A\},\{G C\},\{G T\}$, $\{T A\},\{T C\}$ and $\{T G\}$, are strong comma-free $(S C F)$ by Theorem 2.2. Thus, a dinucleotide unitary strong comma-free code $\left\{l_{1} l_{2}\right\}$ has a permuted code $\left\{l_{2} l_{1}\right\}$ which is also strong comma-free ( $S C F^{2}$ property, see Definition 2.10). Furthermore, the four dinucleotide unitary strong comma-free codes $\{A T\},\{C G\},\{G C\}$ and $\{T A\}$ are self- complementary.

### 2.3.2.2 Unitary circular codes of trinucleotides

The 24 trinucleotide unitary codes $\left\{l_{1} l_{2} l_{3}\right\},\left\{\mathcal{P}\left(l_{1} l_{2} l_{3}\right)\right\}=\left\{l_{2} l_{3} l_{1}\right\}$ and $\left\{\mathcal{P}^{2}\left(l_{1} l_{2} l_{3}\right)\right\}$ $=\left\{l_{3} l_{1} l_{2}\right\}$ with $l_{1}, l_{2}, l_{3} \in B$ and $l_{1} \neq l_{2} \neq l_{3}$, i.e. $\{A C G\},\{A C T\},\{A G C\}$, $\{A G T\},\{A T C\},\{A T G\},\{C A G\},\{C A T\},\{C G A\},\{C G T\},\{C T A\},\{C T G\}$, $\{G A C\},\{G A T\},\{G C A\},\{G C T\},\{G T A\},\{G T C\},\{T A C\},\{T A G\},\{T C A\}$, $\{T C G\},\{T G A\}$ and $\{T G C\}$, are strong comma-free $(S C F)$ by Theorem 2.2. Thus, a trinucleotide unitary strong comma-free code $\left\{l_{1} l_{2} l_{3}\right\}$ has two permuted codes $\left\{l_{2} l_{3} l_{1}\right\}$ and $\left\{l_{3} l_{1} l_{2}\right\}$ which are also strong comma-free (SCF ${ }^{3}$ property, see Definition 2.10).

The 24 trinucleotide unitary codes $\left\{l_{1} l_{1} l_{2}\right\}$ and $\left\{\mathcal{P}^{2}\left(l_{1} l_{1} l_{2}\right)\right\}=\left\{l_{2} l_{1} l_{1}\right\}$ with $l_{1}, l_{2} \in B$ and $l_{1} \neq l_{2}$, i.e. $\{A A C\},\{A A G\},\{A A T\},\{A C C\},\{A G G\}$, $\{A T T\},\{C A A\},\{C C A\},\{C C G\},\{C C T\},\{C G G\},\{C T T\},\{G A A\},\{G C C\}$, $\{G G A\},\{G G C\},\{G G T\},\{G T T\},\{T A A\},\{T C C\},\{T G G\},\{T T A\},\{T T C\}$ and $\{T T G\}$, are strong comma-free $(S C F)$ by Theorem 2.2. The 12 trinucleotide unitary codes $\left\{\mathcal{P}\left(l_{1} l_{1} l_{2}\right)\right\}=\left\{l_{1} l_{2} l_{1}\right\}$ with $l_{1}, l_{2} \in B$ and $l_{1} \neq l_{2}$, i.e. $\{A C A\}$, $\{A G A\},\{A T A\},\{C A C\},\{C G C\},\{C T C\},\{G A G\},\{G C G\},\{G T G\},\{T A T\}$, $\{T C T\}$ and $\{T G T\}$, are comma-free $(C F)$ by Theorem 2.1. Thus, a trinucleotide unitary strong comma-free code $\left\{l_{1} l_{1} l_{2}\right\}$ has one permuted code $\left\{l_{2} l_{1} l_{1}\right\}$ which is also strong comma-free and one permuted code $\left\{l_{1} l_{2} l_{1}\right\}$ which is commafree code. Corollary, a trinucleotide unitary comma-free code $\left\{l_{1} l_{2} l_{1}\right\}$ has two permuted codes $\left\{l_{1} l_{1} l_{2}\right\}$ and $\left\{l_{2} l_{1} l_{1}\right\}$ which are strong comma-free.

### 2.3.2.3 Unitary circular codes of tetranucleotides

The 24 tetranucleotide unitary codes $\left\{l_{1} l_{2} l_{3} l_{4}\right\},\left\{\mathcal{P}\left(l_{1} l_{2} l_{3} l_{4}\right)\right\}=\left\{l_{2} l_{3} l_{4} l_{1}\right\}$, $\left\{\mathcal{P}^{2}\left(l_{1} l_{2} l_{3} l_{4}\right)\right\}=\left\{l_{3} l_{4} l_{1} l_{2}\right\}$ and $\left\{\mathcal{P}^{3}\left(l_{1} l_{2} l_{3} l_{4}\right)\right\}=\left\{l_{4} l_{1} l_{2} l_{3}\right\}$ with $l_{1}, l_{2}, l_{3}, l_{4} \in B$ and $l_{1} \neq l_{2} \neq l_{3} \neq l_{4}$ are strong comma-free (SCF) by Theorem 2.2. Thus, a tetranucleotide unitary strong comma-free code $\left\{l_{1} l_{2} l_{3} l_{4}\right\}$ has three permuted codes $\left\{l_{2} l_{3} l_{4} l_{1}\right\},\left\{l_{3} l_{4} l_{1} l_{2}\right\}$ and $\left\{l_{4} l_{1} l_{2} l_{3}\right\}$ which are also strong comma-free $\left(S C F^{4}\right.$ property, see Definition 2.1). The 48 tetranucleotide unitary codes $\left\{l_{1} l_{2} l_{1} l_{3}\right\}$, $\left\{\mathcal{P}\left(l_{1} l_{2} l_{1} l_{3}\right)\right\}=\left\{l_{2} l_{1} l_{3} l_{1}\right\},\left\{\mathcal{P}^{2}\left(l_{1} l_{2} l_{1} l_{3}\right)\right\}=\left\{l_{1} l_{3} l_{1} l_{2}\right\}$ and $\left\{\mathcal{P}^{3}\left(l_{1} l_{2} l_{1} l_{3}\right)\right\}=\left\{l_{3} l_{1} l_{2} l_{1}\right\}$ with $l_{1}, l_{2}, l_{3} \in B$ and $l_{1} \neq l_{2} \neq l_{3}$ are strong comma-free (SCF) by Theorem 2.2. Thus, a tetranucleotide unitary strong comma-free code $\left\{l_{1} l_{2} l_{1} l_{3}\right\}$ has three permuted codes $\left\{l_{2} l_{1} l_{3} l_{1}\right\},\left\{l_{1} l_{3} l_{1} l_{2}\right\}$ and $\left\{l_{3} l_{1} l_{2} l_{1}\right\}$ which are also strong comma-free ( $S C F^{4}$ property).

The 72 tetranucleotide unitary codes $\left\{l_{1} l_{1} l_{2} l_{3}\right\},\left\{\mathcal{P}^{2}\left(l_{1} l_{1} l_{2} l_{3}\right)\right\}=\left\{l_{2} l_{3} l_{1} l_{1}\right\}$ and
$\left\{\mathcal{P}^{3}\left(l_{1} l_{1} l_{2} l_{3}\right)\right\}=\left\{l_{3} l_{1} l_{1} l_{2}\right\}$ with $l_{1}, l_{2}, l_{3} \in B$ and $l_{1} \neq l_{2} \neq l_{3}$ are strong commafree $(S C F)$ by Theorem 2.2. The 24 tetranucleotide unitary codes $\left\{\mathcal{P}\left(l_{1} l_{1} l_{2} l_{3}\right)\right\}$ $=\left\{l_{1} l_{2} l_{3} l_{1}\right\}$ with $l_{1}, l_{2}, l_{3} \in B$ and $l_{1} \neq l_{2} \neq l_{3}$ are comma-free $(C F)$ by Theorem 2.1. Thus, a tetranucleotide unitary strong comma-free code $\left\{l_{1} l_{1} l_{2} l_{3}\right\}$ has two permuted codes $\left\{l_{2} l_{3} l_{1} l_{1}\right\}$ and $\left\{l_{3} l_{1} l_{1} l_{2}\right\}$ which are also strong comma-free and one permuted code $\left\{l_{1} l_{2} l_{3} l_{1}\right\}$ which is comma-free. Corollary, a tetranucleotide unitary comma-free code $\left\{l_{1} l_{2} l_{3} l_{1}\right\}$ has three permuted codes $\left\{l_{1} l_{1} l_{2} l_{3}\right\}$, $\left\{l_{2} l_{3} l_{1} l_{1}\right\}$ and $\left\{l_{3} l_{1} l_{1} l_{2}\right\}$ which are strong comma-free.

The 24 tetranucleotide unitary codes $\left\{l_{1} l_{1} l_{1} l_{2}\right\}$ and $\left\{\mathcal{P}^{3}\left(l_{1} l_{1} l_{1} l_{2}\right)\right\}=\left\{l_{2} l_{1} l_{1} l_{1}\right\}$ with $l_{1}, l_{2} \in B$ and $l_{1} \neq l_{2}$ are strong comma-free $(S C F)$ by Theorem 2.2. The 24 tetranucleotide unitary codes $\left\{\mathcal{P}\left(l_{1} l_{1} l_{1} l_{2}\right)\right\}=\left\{l_{1} l_{1} l_{2} l_{1}\right\}$ and $\left\{\mathcal{P}^{2}\left(l_{1} l_{1} l_{1} l_{2}\right)\right\}=$ $\left\{l_{1} l_{2} l_{1} l_{1}\right\}$ with $l_{1}, l_{2} \in B$ and $l_{1} \neq l_{2}$ are comma-free (CF) by Theorem 2.1. Thus, a tetranucleotide unitary strong comma-free code $\left\{l_{1} l_{1} l_{1} l_{2}\right\}$ has one permuted code $\left\{l_{2} l_{1} l_{1} l_{1}\right\}$ which is also strong comma-free and two permuted codes $\left\{l_{1} l_{1} l_{2} l_{1}\right\}$ and $\left\{l_{1} l_{2} l_{1} l_{1}\right\}$ which are comma-free. Corollary, a tetranucleotide unitary commafree code $\left\{l_{1} l_{1} l_{2} l_{1}\right\}$ has one permuted code $\left\{l_{1} l_{2} l_{1} l_{1}\right\}$ which is also comma-free and two permuted codes $\left\{l_{1} l_{1} l_{1} l_{2}\right\}$ and $\left\{l_{2} l_{1} l_{1} l_{1}\right\}$ which are strong comma-free. The 12 tetranucleotide unitary codes $\left\{l_{1} l_{1} l_{2} l_{2}\right\}$ and $\left\{\mathcal{P}^{2}\left(l_{1} l_{1} l_{2} l_{2}\right)\right\}=\left\{l_{2} l_{2} l_{1} l_{1}\right\}$ with $l_{1}, l_{2} \in B$ and $l_{1} \neq l_{2}$ are strong comma-free $(S C F)$ by Theorem 2.2.

The 12 tetranucleotide unitary codes $\left\{\mathcal{P}\left(l_{1} l_{1} l_{2} l_{2}\right)\right\}=\left\{l_{1} l_{2} l_{2} l_{1}\right\}$ and $\left\{\mathcal{P}^{3}\left(l_{1} l_{1} l_{2} l_{2}\right)\right\}=\left\{l_{2} l_{1} l_{1} l_{2}\right\}$ with $l_{1}, l_{2} \in B$ and $l_{1} \neq l_{2}$ are comma-free (CF) by Theorem 2.1. Thus, a tetranucleotide unitary strong comma-free code $\left\{l_{1} l_{1} l_{2} l_{2}\right\}$ has one permuted code $\left\{l_{2} l_{2} l_{1} l_{1}\right\}$ which is also strong comma-free and two permuted codes $\left\{l_{1} l_{2} l_{2} l_{1}\right\}$ and $\left\{l_{2} l_{1} l_{1} l_{2}\right\}$ which are comma-free. Corollary, a tetranucleotide unitary comma-free code $\left\{l_{1} l_{2} l_{2} l_{1}\right\}$ has one permuted code $\left\{l_{2} l_{1} l_{1} l_{2}\right\}$ which is also comma-free and two permuted codes $\left\{l_{1} l_{1} l_{2} l_{2}\right\}$ and $\left\{l_{2} l_{2} l_{1} l_{1}\right\}$ which are strong comma-free.

Furthermore, the 12 tetranucleotide unitary strong comma-free codes $\{A A T T\},\{A C G T\},\{A G C T\},\{C A T G\},\{C C G G\},\{C T A G\},\{G A T C\}$, $\{G G C C\},\{G T A C\},\{T C G A\},\{T G C A\}$ and $\{T T A A\}$ are self-complementary. There is no tetranucleotide unitary comma-free code which is self-complementary.

### 2.4 Summary

We briefly gave the history of the genetic code, starting from its root with the discovery of the structure of DNA, through the various code theories, such as diamond code and comma-free code, that were proposed to help solve the mystery of nucleotides coding for amino acids. This shows us how circular code was discovered in 1996, why it is still relevant and an interesting study subject even though the genetic code has been already discovered and proven. We presented the circular code with its features and properties. We defined the unitary circular code, explaining how simple repeats are circular code. This allows us to study the large amount of sequences that has low-complexity DNA.

In the next chapter we will present the data that was retrieved for ribosomal RNA and eukaryotic genomes. For each environment we developed different algorithms to extract the results we need. Furthermore, several methods were adopted to help use analyse the results obtained. For this we divided our context into: (i) $X$ circular code motifs in ribosomal RNA, (ii) $X$ circular code motifs in eukaryotic genomes (iii) unitary circular codes (simple repeats) in eukaryotic genomes and (iv) trinucleotide pairs in gene sequences.

## 3

## Data and Methods

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### 3.1 Introduction

Previously, we explained the biological context in which we are working and the type of data needed for our study. Now we will show the actual data, how it was obtained, processed and dealt with it.

An extensive collection of classes and algorithms were written to handle our data, from reading PDB, alignments and huge FastA files, to searching sequences for motifs, all the way to the statistical tests applied to the found motifs. Our code has been refined and optimized to handle huge flat files with fast processing time.

We grouped together the search algorithms and statistical analysis tools depending on topic. We have a group that handles $X$ circular code motifs and its comparison with various other codes. While another group aim at the study of unitary circular codes, otherwise known as simple repeats. Two sections handle the tools used for ribosome examination and another that studies the trinucleotide pairing in gene sequences.

### 3.2 DATA ACQUISITION

The used in this work can be divided along two different studies. The first focused on spatial interaction of circular code motifs, for which it required structural data of ribosomes from PDB archive. The second study concerned the search of circular code motifs on a genomic scale, which were retrieved from RefSeq database.

### 3.2.1 Ribosomal data

To study structural significance of circular code motifs, we collected ribosomal data from the before mentioned PDB archive in 2014 (Section 1.4.1, www.rcsb. org/pdb). The selected PDB entries have necessarily a bacterial 16 S rRNA or a eukaryotic 18 S rRNA. An entry from each organism having this criteria was chosen, with preference going towards entries having an mRNA and all or any tRNAs. This emphasize allows us to better study the spatial interaction of $X$ motifs from rRNA, mRNA and tRNA (Section 4.2).

The studied PDB crystallographic structures are two bacterial entries: Escherichia coli (Brilot et al., 2013) and Thermus thermophilus (Jenner et al., 2010); one archaea: Pyrococcus furiosus (Armache et al., 2013); three nuclear eukaryotes: Saccharomyces cerevisiae (Armache et al., 2010a), Triticum aes-

Table 3.1: $X$ circular code motifs studied in seven crystallographic structures of the Protein Data Bank PDB (www.rcsb.org/pdb). The main features of the studied crystallographic structures are given: PDB identification, kingdom, organism, type (16S for prokaryotes, 18S for eukaryotes) and base length (b) of rRNA, mRNA (Yes for available, No for unavailable), location of tRNA for the A, P, and E sites (No for unavailable).

|  |  |  |  | tRNAs |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PDB ID | Kingdom | Organism | rRNA | mRNA | A | P | E |
| 3J5T | Bacteria | Escherichia <br> coli | 16 S <br> $(1542 \mathrm{~b})$ | Yes | Phe | Phe | No |
| 3I8G | Bacteria | Thermus <br> thermophilus | 16 S <br> $(1516 ~ b)$ | Yes | Phe | Phe | Phe |
| 3J20 | Archaea | Pyrococcus <br> furiosus | 16 S <br> $(1495 \mathrm{~b})$ | No | No | Phe | Phe |
| 3IZE | Eukaryote | Saccharomyces <br> cerevisiaie | 18 S <br> $(1800 \mathrm{~b})$ | Yes | No | Asp | No |
| 3J5Z | Eukaryote | Triticum <br> aestivum | 18 S <br> $(1810 \mathrm{~b})$ | Yes | No | Asp | No |
| 3J3D | Eukaryote | Homo <br> Sapiens | 18 S <br> $(1869 \mathrm{~b})$ | Yes | No | No | Met |
| 3BBN | Eukaryote | Spinacia <br> oleracea | 16 S | No | No | No | No |

tivum (Armache et al., 2010a,b; Gogala et al., 2014) and Homo sapiens (Anger et al., 2013); finally one chloroplast (eukaryotic organelle): Spinacia oleracea (Sharma et al., 2007).

### 3.2.2 Genomic database

Using BioPerl, we were able to retrieve all the eukaryotic chromosome sequences from the RefSeq database (Reference Sequence). The RefSeq is a curated nonredundant sequence database of genomes. We took one species from each genus and only complete genomic molecules (designated as NC), excluding alternate assembly. One strain from each species is considered.

Genomes with total size less than 400000 bases were filtered out (to avoid data results with null value). The following six genomes were dropped due to this: Cryptomonas paramecium (size $=82348$ bases), Encephalitozoon cuniculi (size $=357485$ bases), Encephalitozoon hellem (size $=245811$ bases), Encephalitozoon intestinalis (size $=230782$ bases), Encephalitozoon romaleae (size $=215619$ bases) and Nitzschia (size $=14661$ bases).

After this we retrieved the Genbank file associated with each chromosome which allowed us to extract the coordinates of the coding regions (CDS) mapped on that chromosome.

The outcome of this is 138 eukaryotic genomes (Sections 4.3, 4.4 and 4.5), displayed in Table 3.2. This genome information represents a total of $91,421,182,030$ bases with $3,133,622,680$ bases in coding regions ( $3.4 \%$ ) and $88,287,559,350$ bases for the non-coding regions ( $96.6 \%$ ).

Table 3.2: Genomes characteristics.

| Organism | Size (base) | CDS | non-CDS | Organism | Size | CDS | non-CDS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anolis carolinensis | 1081644591 | 16670366 | 1064974225 | Microtus ochrogaster | 1655383507 | 23979075 | 1631404432 |
| Anopheles gambiae | 24393108 | 1935976 | 22457132 | Monodelphis domestica | 2754317877 | 26405867 | 2727912010 |
| Apis mellifera | 219629612 | 16592730 | 203036882 | Mus musculus | 1205572488 | 14915903 | 1190656585 |
| Arabidopsis thaliana | 119146348 | 33175579 | 85970769 | Myceliophthora thermophila | 16385300 | 5976840 | 10408460 |
| Aspergillus fumigatus | 29384958 | 14214225 | 15170733 | Nasonia vitripennis | 116029644 | 13849880 | 102179764 |
| Babesia bigemina | 10271324 | 6801553 | 3469771 | Naumovozyma castellii | 11219539 | 8316040 | 2903499 |
| Babesia bovis | 4322739 | 2942868 | 1379871 | Naumovozyma dairenensis | 13527580 | 8618630 | 4908950 |
| Babesia microti | 6381289 | 4627831 | 1753458 | Callithrix jacchus | 2770219215 | 32941100 | 2737278115 |
| Beta vulgaris | 376583697 | 24577029 | 352006668 | Cyanidioschyzon merolae | 16546747 | 7429255 | 9117492 |
| Bombus terrestris | 216849342 | 16328176 | 200521166 | Esox lucius | 701024151 | 36362423 | 664661728 |
| Bos taurus | 2715765904 | 34037257 | 2681728647 | Leishmania mexicana | 30937689 | 15190689 | 15747000 |
| Brachypodium did | 271776478 | 33348761 | 238427717 | Neospora caninum | 57547420 | 17793122 | 39754298 |
| Brassica napus | 775113993 | 94788198 | 680325795 | Phaeodactylum tricornutum | 26138756 | 13966979 | 12171777 |
| Brassica oleracea | 446885882 | 51062958 | 395822924 | Solanum lycopersicum | 802138220 | 33641774 | 768496446 |
| Brassica rapa | 256423463 | 48059517 | 208363946 | Neurospora crassa | 40463072 | 14868399 | 25594673 |
| Caenorhabditis briggsae | 91234787 | 20457533 | 70777254 | Nomascus leucogeny | 2795260045 | 32421857 | 2762838188 |
| Caenorhabditis elegans | 100272607 | 26613936 | 73658671 | Ogataea parapolymorp | 8874589 | 7499949 | 1374640 |
| Camelina sativa | 578444267 | 95232658 | 483211609 | Oreochromis niloticus | 657350972 | 35450942 | 621900030 |
| Candida dubliniensis | 14618422 | 8917936 | 5700486 | Ornithorhynchus anatinus | 437080024 | 4691381 | 432388643 |
| Candida glabrata | 12318245 | 7914961 | 4403284 | Oryctolagus cuniculus | 2247752104 | 24420043 | 2223332061 |
| Candida orthopsilosis | 12659401 | 8468943 | 4190458 | Oryza brachyantha | 250923338 | 28480058 | 222443280 |
| Canis lupus | 2327633984 | 34021609 | 2293612375 | Oryza sativa | 382150945 | 30547069 | 351603876 |
| Capra hircus | 2524662720 | 30265609 | 2494397111 | Oryzias latipes | 723441489 | 33534282 | 689907207 |
| Chlorocebus sabaeu | 2744115311 | 35692308 | 2708423003 | Ostreococcus lucimarinus | 13204888 | 9216998 | 3987890 |
| Chrysemys picta | 461747357 | 5803852 | 455943505 | Ostreococcus tauri | 12456351 | 10138133 | 2318218 |
| Cicer arietinum | 347247377 | 28623811 | 318623566 | Ovis aries | 2584815894 | 33794145 | 2551021749 |
| Ciona intestinalis | 78296155 | 15550846 | 62745309 | Pan paniscus | 3151907227 | 33289497 | 3118617730 |
| Citrus sinensis | 238999708 | 27421823 | 211577885 | Pan troglodytes | 3091112213 | 34403316 | 3056708897 |
| Cryptococcus gattii | 18374760 | 10193549 | 8181211 | Papio anubis | 2724327674 | 34126862 | 2690200812 |
| Cryptococcus neoformans | 19699782 | 10546316 | 9153466 | Phaseolus vulgaris | 514820528 | 34393133 | 480427395 |
| Cryptosporidium parvum | 9102324 | 6820333 | 2281991 | Plasmodium cynomolgi | 22728335 | 9350600 | 13377735 |
| Cucumis sativus | 191859024 | 25366500 | 166492524 | Plasmodium falciparum | 23264338 | 12245290 | 11019048 |
| Cynoglossus semilaevis | 445139357 | 36786472 | 408352885 | Plasmodium knowlesi | 23462187 | 11118740 | 12343447 |
| Danio rerio | 1340430591 | 44231259 | 1296199332 | Plasmodium vivax | 22621071 | 10906305 | 11714766 |
| Debaryomyces hansenii | 12152486 | 9022180 | 3130306 | Poecilia reticulata | 696700953 | 38655401 | 658045552 |
| Dictyostelium discoideum | 33943072 | 20979100 | 12963972 | Pongo abelii | 3029491029 | 32110815 | 2997380214 |
| Drosophila melanogaster | 28557754 | 4239527 | 24318227 | Populus trichocarpa | 378545565 | 44068914 | 334476651 |
| Drosophila pseudoobscura | 50607275 | 9775205 | 40832070 | Prunus mume | 198852406 | 29298313 | 169554093 |
| Drosophila simulans | 17992287 | 2308887 | 15683400 | Rattus norvegicus | 2782012602 | 37109116 | 2744903486 |
| Drosophila yakuba | 23145337 | 3900892 | 19244445 | Saccharomyces cerevisiae | 12071326 | 8691722 | 3379604 |
| Elaeis guineensis | 657968836 | 27281976 | 630686860 | Salmo salar | 2240204991 | 71170520 | 2169034471 |
| Equus caballus | 2367053447 | 32994722 | 2334058725 | Scheffersomyces stipitis | 15441179 | 8587907 | 6853272 |
| Eremothecium cymbalariae | 9669424 | 6473618 | 3195806 | Schizosaccharomyces pombe | 12571820 | 7138394 | 5433426 |
| Eremothecium gossypii | 9095748 | 7007631 | 2088117 | Sesamum indicum | 233222381 | 30558151 | 202664230 |
| Felis catus | 2419212910 | 32750934 | 2386461976 | Setaria italica | 401296418 | 35711158 | 365585260 |
| Ficedula albicollis | 1044065291 | 25797019 | 1018268272 | Solanum pennellii | 926426464 | 34535865 | 891890599 |
| Fragaria vesca | 198117109 | 30904815 | 167212294 | Sorghum bicolor | 659229367 | 37431478 | 621797889 |
| Gallus gallus | 1021439028 | 28351491 | 993087537 | Sus scrofa | 2596639456 | 32005814 | 2564633642 |
| Glycine max | 949176042 | 60862926 | 888313116 | Taeniopygia guttata | 1021462940 | 23660215 | 997802725 |
| Gorilla gorilla | 2917687013 | 33405815 | 2884281198 | Takifugu rubripes | 281572362 | 28676957 | 252895405 |
| Gossypium raimondii | 749228090 | 44683789 | 704544301 | Tetrapisispora blattae | 14048593 | 8755400 | 5293193 |
| Homo sapiens | 3088269832 | 35915410 | 3052354422 | Tetrapisispora phaffii | 12100190 | 8034104 | 4066086 |
| Kazachstania africana | 11130140 | 7848851 | 3281289 | Thalassiosira pseudonana | 28733535 | 15615332 | 13118203 |


| Organism | Size | CDS | non-CDS | Organism | Size (base) | CDS | non-CDS |
| :--- | ---: | ---: | ---: | :--- | ---: | ---: | ---: |
| Kluyveromyces lactis | 10689156 | 7388969 | 3300187 | Theileria annulata | 8352520 | 6074113 | 2278407 |
| Komagataella phaffi | 9216378 | 7207175 | 2009203 | Theileria equi | 6015803 | 4155315 | 1860488 |
| Lachancea thermotolerans | 10392862 | 7509690 | 2883172 | Theileria orientalis | 8983596 | 6141721 | 2841875 |
| Leishmania braziliensis | 31238104 | 15200552 | 16037552 | Theileria parva | 4511914 | 3080757 | 1431157 |
| Leishmania donovani | 32444968 | 14635818 | 17809150 | Theobroma cacao | 330456197 | 34445261 | 296010936 |
| Leishmania infantum | 31924853 | 15556104 | 16368749 | Thielavia terrestris | 36912256 | 13614268 | 23297988 |
| Leishmania major | 32855089 | 15710352 | 17144737 | Torulaspora delbrueckii | 9220678 | 7248844 |  |
| Leishmania panamensis | 30688794 | 14542185 | 16146609 | Tribolium castaneum | 1971834 |  |  |
| Lepisosteus oculatus | 891144077 | 31820297 | 859323780 | Trypanosoma brucei | 22148088 | 13292454 | 8855634 |
| Macaca fascicularis | 2871826009 | 34465017 | 2837360992 | Ustilago maydis | 19643891 | 11979357 | 7664534 |
| Macaca mulatta | 2835963390 | 34674220 | 2801289170 | Vigna radiata | 333308464 | 29662798 | 303645666 |
| Magnaporthe oryzae | 40491973 | 16673311 | 23818662 | Vitis vinifera | 426176009 | 31432213 | 394743796 |
| Malus domestica | 526197889 | 36138315 | 490059574 | Yarrowia lipolytica | 20502981 | 9430576 | 11072405 |
| Medicago truncatula | 384466993 | 47725325 | 336741668 | Zea mays | 2059701728 | 44366789 | 2015334939 |
| Meleagris gallopavo | 97203167 | 25172179 | 947030988 | Zygosaccharomyces rouxii | 9764635 | 7436797 | 2327838 |
| Micromonas sp. | 20989326 | 14597320 | 6392006 | Zymoseptoria tritici | 39686251 | 14379863 | 25306388 |

### 3.3 Circular code motifs

### 3.3.1 SEARCH ALGORITHMS

We present here the search algorithms of circular code motifs in a nucleic acid sequence. First, we will present the biinfinite motif, a special case of the circular code motif that is not restricted to complete trinucleotides from a circular code set at the start and end of the motif, i.e. it can include a nucleotide or dinucleotide (prefix or suffix) belonging to a circular code trinucleotide. Second, we will present the approach used in previous works which focused on the usage of sets to determine circular code motifs. Finally, we will present the algorithm used to search for circular code motifs in the studies done in this thesis which approached the search from frames point of view.

### 3.3.1.1 Biinfinite word

We consider the classical case of the window length $\tilde{l}$ for a biinfinite word $\tilde{w}=\ldots l_{-1} l_{0} l_{1} \ldots$ for a circular code $X$. We will take the example from Figure 2.4 for a biinfinite word $\tilde{w}=\ldots A G G T A A T T A C C A G \ldots$ of the common circular code $X$. The first nucleotide of $\tilde{w}, A$, is possibly the 1 st, 2 nd or 3 rd nucleotide of trinucleotide of $X$. By trying the three possible factorizations (frames) $\tilde{w}_{0}, \tilde{w}_{1}$ and $\tilde{w}_{2}$ into trinucleotides of $X$, only one factorization, $\tilde{w}_{1}$, is possible ( $\tilde{w}_{1} \tilde{w}_{2}$ being $\tilde{w}_{0}$ shifted by one and two nucleotides receptively). Thus, the first nucleotide $A$ of $\tilde{w}$ is the 3 rd nucleotide of a trinucleotide of $X$. The factorization of $\tilde{w}_{1}$ leads to having the following trinucleotides $N N A, G G T, A A T, T A C$ and $C A G$ that belong to $X$ ( $N$ designates any appropriate letter of $X$ ). The factorization of $\tilde{w}_{0}$ and $\tilde{w}_{2}$ are not a viable options due to the fact that no trinucleotide
of $X$ starts with the prefix $A G$. This case occurs immediately for $\tilde{w}_{0}$ and after 11 nucleotides for $\tilde{w}_{2}$.

Thus, the unique factorization of $\tilde{w}$ is $\tilde{w}_{1}=\ldots A, G G T, A A T, T A C, C A G, \ldots$ $\tilde{w}$ can be located anywhere in a sequence of $X$, i.e. the sequence of $X$ does not require an initiator codon (or a stop codon) to retrieve the reading frame. The finite word $\tilde{w}_{\alpha}=A G G T A A T T A C C A(\tilde{w}$ without the last nucleotide G$)$, with a length of 12 nucleotides, is ambiguous as it has two factorizations $\tilde{w}_{1}$ and $\tilde{w}_{2}$ into trinucleotides of $X$. The word $\tilde{w}_{\alpha}$ is called ambiguous word of $X$. By definition of a circular code, all the ambiguous words are finite words. We will prove that $\tilde{w}_{\alpha}$, taken as an illustration example here, is one of the four longest of $X$. Then, the window length $\tilde{l}$ to retrieve the construction frame of any biinfinite word of a circular code $Y$ is the letter length of the longest ambiguous word $\tilde{w}_{\alpha}$, plus one letter. Thus, with the common circular code $X, \tilde{l}=12+1=13$ nucleotides. The window lengths $\tilde{l}$ for the trinucleotide circular codes $X_{1}=\mathcal{P}(X)$ and $X_{2}=\mathcal{P}^{2}(X)$ are also equal to $\tilde{l}=13$ nucleotides. In conclusion, the retrieval of the reading frame of the common circular code $X$ needs a window length $\tilde{l}$ of 13 nucleotides in each frame.

### 3.3.1.2 SETS ALGORITHM

A set is a collection of distinct elements without repetition and without order. It is written here with sans serif font, e.g. S. A multiset is a generalization of a set. It is an unordered collection of elements with multiple but finite occurrences of any element. It is written here with calligraphy font, e.g. $\mathcal{S}$. The multiplicity $m_{\mathcal{S}}(e)$ of an element $e$ in a multiset $\mathcal{S}$ is its occurrence number. In our context, and for readability reason, a multiset is represented as follows, e.g. $\mathcal{S}=\{A$ : $\left.m_{\mathcal{S}}(A), C: m_{\mathcal{S}}(C), G: m_{\mathcal{S}}(G), T: m_{\mathcal{S}}(T)\right\}$.

We briefly recall the definitions of intersection and union of multisets. Let $\mathcal{S}$ and $\mathcal{T}$ be two multisets. The union $\mathcal{S} \cup \mathcal{T}$ of $\mathcal{S}$ and $\mathcal{T}$ is the multiset defined by $m_{\mathcal{S} \cup \mathcal{T}}(e)=\max \left(m_{\mathcal{S}}(e), m_{\mathcal{T}}(e)\right)$, i.e. the multiplicity of an element in $\mathcal{S} \cup \mathcal{T}$ is equal to the maximum of the multiplicities of the element in $\mathcal{S}$ and $\mathcal{T}$. For example, if $\mathcal{S}=\{A: 3, G: 1, T: 2\}$ and $\mathcal{T}=\{A: 2, C: 1, G: 2\}$ then $\mathcal{S} \cup \mathcal{T}=\{A: 3, C: 1, G: 2, T: 2\}$. The intersection $\mathcal{S} \cap \mathcal{T}$ of $\mathcal{S}$ and $\mathcal{T}$ is the multiset defined by $m_{\mathcal{S} \cap \mathcal{T}}(e)=\min \left(m_{\mathcal{S}}(e), m_{\mathcal{T}}(e)\right)$, i.e. the multiplicity of an element in $\mathcal{S} \cap \mathcal{T}$ is equal to the minimum of the multiplicities of the element in $\mathcal{S}$ and $\mathcal{T}$. Using the previous example, $\mathcal{S} \cap \mathcal{T}=\{A: 2, G: 1\}$. Finally, a subset S of a multiset $\mathcal{S}$ is called the support of $\mathcal{S}$ if for every element $e$ such that

Algorithm 3.1 The algorithm AmbiguousWordsX gives the ambiguous words of the common circular code $X$ in the shifted frames $(f) 1$ and 2 with a length varying from 1 to 11 nucleotides. Remember that there is no ambiguous word of $X$ with a length $l>12$ nucleotides.

```
Algorithm AmbiguousWordsX
// Ambiguous words of X in frames 1 and 2
1 for l \leftarrow 1 to 11 step +1 do // Nucleotide length l
    // Set W}\mp@subsup{W}{}{l}\mathrm{ of words of }X\mathrm{ at length l
    W
    // Multiset }\mp@subsup{\mathcal{W}}{1}{l}\mathrm{ of words of }X\mathrm{ in frame 1 at length l
    \mathcal{W}
    // Multiset }\mp@subsup{\mathcal{W}}{2}{l}\mathrm{ of words of }X\mathrm{ in frame 2 at length l
    \mp@subsup{\mathcal{W}}{2}{l}}\leftarrow\mathrm{ wordsXFrame2 (l)
    for f}\leftarrow1\mathrm{ to 2 step +1 do // Frame f
        // Set M}\mp@subsup{M}{f}{l}\mathrm{ of ambiguous words of }X\mathrm{ in frame }f\mathrm{ at length l
        Mf}\mp@subsup{|}{f}{L}\leftarrow\mp@subsup{W}{}{l}\cap\mp@subsup{\mathcal{W}}{f}{l
        // Multiset }\mp@subsup{\mathcal{M}}{f}{l}\mathrm{ of ambiguous words of }X\mathrm{ in frame }f\mathrm{ at length l
        \mathcal{M}
7
wordsX(l)
// Determination of the set }W\mathrm{ of words of }
    W\leftarrow{}
    if l=1[3] then W\leftarrow X \\frac{l}{3}]}\cdot\mp@subsup{S}{1}{
    else if l=2[3] then W\leftarrowX X [\frac{l}{3}]\cdot\mp@subsup{S}{12}{}
            else }W\leftarrow\mp@subsup{X}{}{\frac{l}{3}]
        return W
wordsXFrame1 (l)
// Determination of the multiset }\mathcal{W}\mathrm{ of words of X in frame 1
        \mathcal{W}}\leftarrow{
        if}l=1\mathrm{ then }\mathcal{W}\leftarrow\mp@subsup{\delta}{2}{
        else if l=2[3] then }\mathcal{W}\leftarrow\mp@subsup{S}{23}{}\cdot\mp@subsup{X}{}{[\frac{l}{3}]
            else if l=0[3] then }\mathcal{W}\leftarrow\mp@subsup{\mathcal{S}}{23}{}\cdot\mp@subsup{X}{}{\frac{l}{3}]-1}\cdot\mp@subsup{\mathcal{S}}{1}{
            else}\mathcal{W}\leftarrow\mp@subsup{\mathcal{S}}{23}{}\cdot\mp@subsup{X}{}{[\frac{l}{3}]-1}\cdot\mp@subsup{S}{12}{
        return W
wordsXFrame2(l)
// Determination of the multiset }\mathcal{W}\mathrm{ of words of X in frame 2
    W}\leftarrow{
    if l=1[3] then }\mathcal{W}\leftarrow\mp@subsup{\delta}{3}{}\cdot\mp@subsup{X}{}{\frac{l}{3}}
    else if l=2[3] then }\mathcal{W}\leftarrow\mp@subsup{\mathcal{S}}{3}{}\cdot\mp@subsup{X}{}{[\frac{l}{3}]}\cdot\mp@subsup{\mathcal{S}}{1}{
            else }\mathcal{W}\leftarrow\mp@subsup{\mathcal{S}}{3}{}\cdot\mp@subsup{X}{}{l\frac{l}{3}]}\cdot\mp@subsup{S}{12}{
    return W\mathcal{L}
```

$m_{\mathcal{S}}(e)>0$ this implies that $e \in \mathbf{S}$, and for every element $e$ such that $m_{\mathcal{S}}(e)=0$ this implies that $e \notin \mathrm{~S}$. For example, the set $\mathrm{S}=\{A, G, T\}$ is the support of $\mathcal{S}=\{A: 3, G: 1, T: 2\}$. For simplification in the writing of the algorithm, the
same operators of intersection and union are used for sets and multisets. Thus, the intersection $\mathcal{S} \cap \mathcal{T}$ of a set S and a multiset $\mathcal{T}$ leads to a set (the support T of $\mathcal{T}$ replacing $\mathcal{T})$. The union $\mathcal{S} \cup \mathcal{T}$ of a set S and a multiset $\mathrm{S}_{2}$ leads to a set (the multiset $\mathcal{S}$ of multiplicity 1 replacing s ). We recall the circular code $X$ :

$$
\begin{aligned}
X= & \{A A C, A A T, A C C, A T C, A T T, C A G, C T C, C T G, G A A, G A C, \\
& G A G, G A T, G C C, G G C, G G T, G T A, G T C, G T T, T A C, T T C\} .
\end{aligned}
$$

Let a word of circular code set $X$ be the three letters $l_{1} l_{2} l_{3}$. Let $\mathrm{S}_{1}$ be the set containing the letters $l_{1}$ of $X, \mathrm{~S}_{2}$ and $\mathrm{S}_{3}$ for $l_{2}$ and $l_{3}$ respectively. Then, $\left(\mathrm{s}_{1}=\mathrm{s}_{2}=\mathrm{s}_{3}=B=\{A, C, G, T\}\right.$.

Let $\mathcal{S}_{1}$ be the multiset containing letters $l_{1}$ of $X, \mathcal{S}_{2}$ and $\mathcal{S}_{3}$ for $l_{2}$ and $l_{3}$ respectively. Then,

$$
\begin{align*}
& \mathcal{S}_{1}=\{A: 5, C: 3, G: 10, T: 2\} \\
& \mathcal{S}_{2}=\{A: 8, C: 2, G: 2, T: 8\}  \tag{3.1}\\
& \mathcal{S}_{3}=\{A: 2, C: 10, G: 3, T: 5\} .
\end{align*}
$$

Remark 3.1. As $X$ is self-complementary, $\mathcal{C}\left(\mathcal{S}_{1}\right)=\{\mathcal{C}(A): 5, \mathcal{C}(C): 3, \mathcal{C}(G)$ : $10, \mathcal{C}(T): 2\}=\{T: 5, G: 3, C: 10, A: 2\}=\mathcal{S}_{3}$. Similarly, $\mathcal{C}\left(\mathcal{S}_{3}\right)=\{\mathcal{C}(A):$ $8, \mathcal{C}(C): 2, \mathcal{C}(G): 2, \mathcal{C}(T): 8\}=\{T: 8, G: 2, C: 2, A: 8\}=\mathcal{S}_{2}$.

Let $\mathrm{S}_{12}$ be the set containing the prefix $l_{1} l_{2}$ of $X, \mathcal{S}_{12}$ the respective multiset. Then,

$$
\begin{align*}
\mathrm{S}_{12}= & \{A A, A C, A T, C A, C T, G A, G C, G G, G T, T A, T T\} \\
\mathcal{S}_{12}= & \{A A: 2, A C: 1, A T: 2, C A: 1, C T: 2,  \tag{3.2}\\
& G A: 4, G C: 1, G G: 2, G T: 3, T A: 1, T T: 1\}
\end{align*}
$$

Let $\mathrm{S}_{23}$ be the set containing the suffix $l_{2} l_{3}$ of $X, \mathcal{S}_{23}$ the respective multiset. Then,

$$
\begin{align*}
\mathrm{S}_{23}= & \{A A, A C, A G, A T, C C, G C, G T, T A, T C, T G, T T\} \\
\mathcal{S}_{23}= & \{A A: 1, A C: 3, A G: 2, A T: 2, C C: 2,  \tag{3.3}\\
& G C: 1, G T: 1, T A: 1, T C: 4, T G: 1, T T: 2\} .
\end{align*}
$$

Remark 3.2. $\operatorname{card}\left(\mathrm{S}_{12}\right)=\operatorname{card}\left(\mathrm{S}_{23}\right)=11$ (among 16 dinucleotides). $\mathrm{S}_{12} \neq \mathrm{S}_{23}$
and $\mathrm{s}_{12} \cap \mathrm{~s}_{23}=\{A A, A C, A T, G C, G T, T A, T T\} . \mathcal{C}\left(\mathrm{s}_{12}\right)=\mathrm{s}_{23}, \mathcal{C}\left(\mathrm{~s}_{23}\right)=\mathrm{s}_{12}$ and $\mathcal{C}\left(\mathrm{s}_{12} \cap \mathrm{~S}_{23}\right)=\mathrm{S}_{23} \cap \mathrm{~S}_{23}$.

Let $A$ and $B$ be two sets of words. $A \cdot B$ is the set of words which are the product (concatenation) of one word of $A$ and one word of $B$, i.e. $A \cdot B=$ $\left\{a_{i} \cdot b_{j} \mid a_{i} \in A, b_{j} \in B\right\}$. Thus, $A^{n}=\underbrace{A \cdot A \cdot \ldots \cdot A}_{n}$ is the set of words which are the product of $n, n \leq 0$, words of A, i.e. $A^{n}=\left\{a_{1} \cdot a_{2} \cdot \ldots \cdot a_{n} \mid a_{i} \in A\right\} A^{0}$ being the empty set. For example, $X^{2}$ is the set of all concatenations of two words of $X$, i.e. $\{A A C A A C, A A C A A T, \ldots, T T C T T C\}$. All these definitions on sets are naturally extended on multisets.

```
Algorithm 3.2 The frames algorithm can retrieve all circular code motifs from
all the frames of a sequence. It is also able to retrieve motifs from other codes,
such as Random codes (Section 4.3.1).
1. Read sequence
2. INIT X AS the set of circular code trinucleotides
3. INIT minsize AS the minimum size of motifs
4. INIT shift
5. FOR EACH frame
6. CASE frame OF
7. 0 : set shift to 0
8. 1 : set shift to 2
9. 2 : set shift to 1
10. ENDCASE
11. INIT motif AS empty
12. FOR EACH trinucleotide in the sequence beginning from
    shift AS tri
        IF X contains tri THEN
                        IF motif is empty THEN
                            Set motif to tri
                ELSE
                    Concatenate tri to motif
                ENDIF
            ELSEIF motif length is larger than minsize THEN
                    Add motif to list of motifs
                Set motif to empty
            ELSE
                Set motif to empty
            ENDIF
        ENDFOR
        ENDFOR
```


### 3.3.1.3 Frames algorithm

Let S be a sequence of nucleotides over $B=\{A, C, G, T\}$. Let $|\mathrm{S}|$ be the number of nucleotides in sequence $\mathbf{S}$. Let Tri be a trinucleotide of $\mathbf{S}$ such that Tri $=$ $l_{i-2} l_{i-1} l_{i} \in A_{4}^{3}, i \leq|\mathrm{S}|$. We can express S as a concatenation of trinucleotides.

With respect to the start of the sequence, the concatenation for frame 0 is $\mathrm{S}_{0}=\operatorname{Tr} i_{1} \cdot \operatorname{Tr} i_{2} \cdot \ldots \cdot \operatorname{Tr} i_{n}, n=\frac{|S|}{3}$ where $\operatorname{Tr} i_{1}$ being the first trinucleotide containing the first three nucleotides $l_{1} l_{2} l_{3}$ of S , Tri $i_{2}$ containing the following nucleotides $l_{4} l_{5} l_{6}$ of S and so on and so forth. Respectively, $\mathrm{S}_{1}=l_{1} \cdot \operatorname{Tr} i_{1} \cdot \operatorname{Tr} i_{2} \cdot \ldots \cdot \operatorname{Tr} i_{n}, n=$ $\frac{|\mathrm{s}|-1}{3}$ and $\mathrm{S}_{2}=l_{1} \cdot l_{2} \cdot \operatorname{Tr} i_{1} \cdot \operatorname{Tr} i_{2} \cdot \ldots \cdot \operatorname{Tr} i_{n}, n=\frac{|\mathrm{s}|-2}{3}$ are the concatenation of trinucleotides of S in frames 1 and 2 .

Let $\mathcal{S}_{0}$ be the multiset containing the trinucleotides $\operatorname{Tr}_{i}$ from $\mathrm{S}_{0}$ with their order respecting their position in the sequence. The frames algorithm will iterate through this multiset matching trinucleotides to those from $X$, e.g. if $\operatorname{Tr} i_{i} \in \mathcal{S}_{0}$, Tri $i_{i}$ will be added to motif $m_{X}$ (motif will start as empty), in case of biinfinite motifs then $\operatorname{Tr} i_{i-1}$ will be examined if $l_{2} l_{3}$ could possibly be part of $X$ motif (as previous algorithm), the algorithm will continue iterating through $\mathcal{S}_{0}$ until it reaches a trinucleotide which does not belong to $X$ which will be inspected if its $l_{1} l_{2}$ belong to $X$ trinucleotides. This process will continue over the whole of $\mathcal{S}_{0}$ till the last trinucleotide, at that point we have collected all possible $X$ circular code motifs in frame 0 , the same will be done for $\mathcal{S}_{1}$ and $\mathcal{S}_{2}$ for frames 1 and 2 respectively.

This approach is faster and allows the maximal retrieval of $X$ circular code motifs in any given sequence regardless of overlapping motifs (for lengths less than 12). The algorithm has four parameters, the sequence to be searched for the motifs, the set used to construct the motifs (which makes the algorithm generic and not restricted to $X$ circular code), the minimum length of a motif (in nucleotides) and minimum number of unique trinucleotides from the given set that a motif should be composed of (used for the retrieval of motifs in sections 4.2 and 4.3).

### 3.3.1.3.1 Definition of a motif

The output of the Frames algorithm is a list of motifs found in all the frames of an examined sequence S using a set $Y$. Each motif $m$ has the following set of attributes: the motif (as a sequence or displayed as trinucleotides with comma delimiter), the index of the first nucleotide in the motif with respect to the
sequence $S$, the index of the last nucleotide in the motif with respect to the sequence $S$, the length of the motif in nucleotides, the frame in which the motif was found with respect to the sequence $\mathbf{S}$, the number of unique trinucleotides present in the motif $m$ according to the set $Y$ and finally the number of unique trinucleotides $(W)$ present in the motif according the $X_{0}$ circular code (Equation 2.2) which will be called trinucleotide cardinality (composition).

In particular, we are interested in the length and cardinality of a motif.

$$
\left\{\begin{array}{l}
n=l(m(Y)) \\
\operatorname{Card}\left(\left\{W_{1}\right\} \cup\left\{W_{2}\right\} \cup \ldots \cup\left\{W_{n}\right\}\right)=\operatorname{Card}(\{W(m(Y))\}) .
\end{array}\right.
$$

The motif class studied in this paper are called large motifs, they are defined by two conditions on their length and cardinality

$$
\left\{\begin{array}{l}
n=l(m(Y)) \geq 15 \text { trinucleotides }  \tag{3.4}\\
\operatorname{Card}(\{W(m(Y))\}) \geq 10 \text { trinucleotides }
\end{array}\right.
$$

These attributes, coupled with the knowledge of what organism we are searching in, allows us to conduct very specific and detailed studies along different angles of a motif. The algorithm in its ability to construct motifs from any given set, which allows to compare the results of $X$ motifs against other sets.

### 3.3.2 Definition of random code motifs

The motifs $m(X), m(\Pi(X)), m\left(X_{1}\right)$ and $m\left(X_{2}\right)$ are generated from the maximal circular codes $X, \Pi(X), X_{1}$ and $X_{2}$, respectively. All these circular codes have 20 trinucleotides with the same total numbers of nucleotides, i.e. $15 A, 15 C$, $15 G, 15 T$. Furthermore, by definition of a circular code, they have neither a periodic trinucleotide $P^{3}=\{A A A, C C C, G G G, T T T\}$ nor two non-periodic permuted trinucleotides $\{t, \mathcal{P}(t)\}$ (Remark 2.1). In order to have an evaluation of the statistical significance of occurrence numbers of the large motifs $m(X)$, $m(\Pi(X)), m\left(X_{1}\right)$ and $m\left(X_{2}\right), 30$ random codes $R$ are generated with respect to the four necessary conditions of maximal circular codes: (i) a random code $R$ with a number of trinucleotides equal to 20; (ii) a random code $R$ without a periodic trinucleotide $P^{3}$; (iii) a random code $R$ without two non-periodic
permuted trinucleotide and (iv) a random code $R$ containing the same total numbers of nucleotides ( $15 A, 15 C, 15 G, 15 T)$. Then, a random code $R$ of trinucleotides randomly chosen in $B^{3}$ is generated satisfying the four conditions (i), (ii), (iii) and (iv).

The complete list of generated random sets are displayed below for reference for future comparison and reproducibility of results:


#### Abstract

$R_{1}=\{A A C, A A T, A C A, A C G, A C T, A G A, C C T, C G A, C T G, G A C, G C A, G C G, G G C, G T C, T A G, T C T, T G A, T G T, T T C, T T G\}$ $R_{2}=\{A C C, A C T, A G C, A T G, A T T, C C A, C T A, G A A, G A G, G C A, G C G, G G C, G T T, T A C, T A G, T A T, T C C, T C G, T G A, T G C\}$ $R_{3}=\{A A G, A A T, A C A, A C T, A G T, A T G, A T T, C A G, C C T, C G G, C T T, G A C, G A T, G C A, G C G, G T C, T C C, T C G, T G A, T G C\}$ $R_{4}=\{A A T, A G A, A G T, A T C, A T G, C A A, C A G, C C G, C G C, C G G, C T A, G A T, G C C, G C T, G T C, T A A, T C T, T G C, T G G, T T A\}$ $R_{5}=\{A A C, A C A, A C C, A G A, A G T, C A A, C C G, C G G, C G T, G C G, G C T, G G T, G T C, G T T, T A A, T A G, T A T, T C A, T C G, T T C\}$ $R_{6}=\{A A C, A C A, A C G, A G G, A T G, C A A, C A T, C C G, C G C, C T T, G A A, G C T, G G C, G T C, G T T, T A G, T C A, T G C, T T A, T T G\}$ $R_{7}=\{A A C, A A T, A C A, A G G, A T A, A T G, C A C, C C A, C G C, G A T, G C C, G G A, G G T, G T C, G T T, T A G, T C C, T C T, T G G, T T C\}$ $R_{8}=\{A A G, A C A, A C C, A T C, A T G, A T T, C A A, C C T, C G G, C T A, C T G, G A T, G C A, G G A, G G T, G T G, T C A, T C C, T G G, T T C\}$ $R_{9}=\{A A C, A C T, A G C, A G T, A T A, A T G, C C G, C G A, C G C, C T T, G A A, G C G, G C T, T A C, T A G, T C A, T C C, T G A, T G G, T G T\}$ $R_{10}=\{A C A, A C C, A G G, A G T, A T T, C A A, C C A, C C T, C G A, C G G, C T A, G A G, G C G, G C T, G T T, T A C, T A G, T C G, T T A, T T G\}$ $R_{11}=\{A A C, A C G, A T A, C A A, C C T, C G C, C T A, C T G, G A C, G C A, G C T, G G A, G T G, G T T, T A A, T A C, T A G, T C T, T G C, T G G\}$ $R_{12}=\{A A C, A C C, A C G, A G C, A T A, C A A, C A C, C G C, C T T, G A T, G C T, G G C, G G T, G T G, T A A, T C A, T G A, T G C, T G T, T T G\}$ $R_{13}=\{A C C, A G C, A T A, A T C, A T G, C A A, C C T, C G G, C T C, G A C, G A T, G C T, G G A, G T A, G T C, G T G, T A A, T C A, T C T, T G G\}$ $R_{14}=\{A A T, A C A, A C G, A C T, A G T, A T A, C A A, C C T, C G C, C T G, G A C, G A T, G C T, G G A, G G T, G T C, T A G, T C C, T G C, T G T\}$ $R_{15}=\{A A C, A C A, A C T, A G T, A T C, A T G, C A C, C A G, C C G, C G C, C T T, G A A, G C G, G G T, G T G, T C A, T C G, T G A, T G T, T T A\}$ $R_{16}=\{A A G, A C A, A G G, A T C, C A G, C A T, C C A, C G A, C G G, C G T, C T A, C T T, G C G, G T A, T A A, T A C, T C G, T G G, T T C, T T G\}$ $R_{17}=\{A A C, A A T, A C A, A C C, A G G, C A T, C C T, C G T, G A A, G A C, G C A, G C G, G G A, G G T, G T T, T C C, T C T, T G C, T T A, T T G\}$ $R_{18}=\{A A C, A A G, A C C, A C G, A G C, A T A, A T T, C A T, C G A, C G G, C T G, G A C, G C A, G C T, G G C, G T T, T C G, T C T, T G T, T T A\}$ $R_{19}=\{A A C, A C C, A C T, A G G, A T A, A T C, C A G, C G A, C G T, C T C, C T T, G A G, G C T, G G A, G G C, T A C, T A T, T C T, T G A, T G G\}$ $R_{20}=\{A A T, A C C, A C T, A G T, A T T, C A C, C C G, C G G, C G T, C T G, G A A, G A T, G C C, G G C, G T A, G T C, T A A, T A T, T C G, T G A\}$ $R_{21}=\{A C G, C A C, C C A, C C G, C T A, C T C, G A A, G A G, G A T, G C A, G C G, G T A, G T G, T A A, T C A, T G A, T G C, T G T, T T A, T T C\}$ $R_{22}=\{A A G, A A T, A C A, A C T, A G C, A T C, A T T, C C G, C C T, C G C, C G G, G A C, G A G, G A T, G G C, G G T, T A T, T G C, T T A, T T C\}$ $R_{23}=\{A A C, A G T, A T G, C A A, C C A, C C T, C G A, C G G, C T A, C T C, C T G, G A T, G C A, G G A, G G T, T A G, T C A, T C T, T G G, T T A\}$ $R_{24}=\{A A G, A C G, A T A, A T C, C A A, C A G, C A T, C C G, C G T, C T C, C T G, G A T, G G C, G G T, G T A, G T G, T A C, T C A, T C G, T T A\}$ $R_{25}=\{A A C, A C C, A T A, A T G, C A G, C G A, C G T, C T A, C T T, G A C, G C A, G C G, G G A, G G T, G T T, T A A, T A C, T C C, T G C, T T G\}$ $R_{26}=\{A C A, A G A, A G G, C A C, C C A, C C G, C T A, C T C, C T G, G A A, G A T, G C T, G G A, G T G, T A G, T C A, T C T, T G G, T T A, T T C\}$ $R_{27}=\{A A T, A T G, C A A, C A C, C A G, C C A, C G G, C T A, C T T, G A C, G C C, G G A, G G C, G G T, T A G, T A T, T C T, T G A, T G C, T T A\}$ $R_{28}=\{A A G, A A T, A C A, A C C, A G A, A T G, C A T, C T G, G A C, G C C, G C T, G G A, G G T, G T T, T A C, T C C, T C G, T G A, T G C, T T C\}$ $R_{29}=\{A A T, A C A, A C G, A T G, A T T, C A C, C C G, C C T, C G A, C T C, G A A, G C C, G G A, G G T, G T A, G T C, G T T, T A G, T A T, T C G\}$ $R_{30}=\{A A G, A C A, A C T, A G A, A G T, A T A, A T T, C C G, C G A, C G C, C T C, G A C, G C C, G G C, G T G, T A A, T G C, T G T, T T C, T T G\}$


### 3.3.3 Definition of the 23 BiJective circular codes motifs

### 3.3.3.1 BiJective transformation circular codes

There are 23 bijective transformation circular codes $\Pi(X)=\left\{\pi_{1}(X), \ldots, \pi_{23}(X)\right\}$ of the maximal $C^{3}$ self-complementary trinucleotide circular code $X=\pi_{0}(X)$. The notation of bijective transformations used here is based on the notation of (Michel and Seligmann, 2014) which relies on (i) the transcript data identified
from the human mitochondrial genomes by (Seligmann, 2013a,b); (ii) the biological function of the polymerase. These biological observations suggest that bijective transformations of RNA transcripts using only two bases are simpler than bijective transformations of three bases which are also simpler than bijective transformations of four bases. Another notation of bijective transformations of circular codes is also proposed by Fimmel, Danielli, and Strüngmann, 2013 in a study of circular codes based on group theory.

### 3.3.3.1.1 Symmetric and asymmetric transformation

The 23 bijective transformation circular codes $\Pi(X)$ of $X$ can be partitioned into nine symmetric bijective transformation circular codes $\Pi_{\mathcal{S}}(X)=$ $\left\{\pi_{1}(X), \ldots, \pi_{9}(X)\right\}$ and 14 asymmetric bijective transformation circular codes $\Pi_{\mathcal{A}}(X)=\left\{\pi_{10}(X), \ldots, \pi_{23}(X)\right\}$ (Table 3.3). The number $N(n, p)$ of bijective transformation circular codes at $p$ letters among $n$ letters is equal to $N(n, p)=\frac{n!}{(n-p)!p}$.

The nine symmetric bijective transformation circular codes $\Pi_{\mathcal{S}}(X)$ can be partitioned into:

1. $N(4,2)=6$ symmetric bijective transformation circular codes $\Pi_{\mathcal{S}, 2}(X)$ at 2 letters.

$$
\begin{aligned}
\Pi_{\mathcal{S}, 2}(X)= & \left\{\pi_{1}(X):(A, C), \pi_{2}(X):(A, G), \pi_{3}(X):(A, T),\right. \\
& \left.\pi_{4}(X):(C, G), \pi_{5}(X):(C, T), \pi_{6}(X):(G, T)\right\}
\end{aligned}
$$

where $\pi_{i}(X):\left(l_{1}, l_{2}\right)$ is the $i$ th bijective transformation in the lexicographical order of the letter $l_{1} \in B$ into the letter $l_{2} \in B, l_{2} \neq l_{1}$, and reciprocally.
2. $\frac{N(4,2)}{2}=3$ symmetric bijective transformation circular codes $\Pi_{\mathcal{S}, 2,2(X)}$ of two disjoint transformations at 2 letters.

$$
\Pi_{\mathcal{S}, 2,2}(X)=\left\{\pi_{7}(X), \pi_{8}(X), \pi_{9}(X)\right\}
$$

where $\pi_{i}(X):\left(l_{1}, l_{2}\right)\left(l_{3}, l_{4}\right)$ is the $i$ th bijective transformation in the lexicographical order of the letter $l_{1} \in B$ into the letter $l_{2} \in B, l_{2} \neq l_{1}$, and reciprocally, and of the letter $l_{3} \in B, l_{3} \neq l_{2} \neq l_{1}$, into the letter $l_{4} \in B$, $l_{4} \neq l_{3} \neq l_{2} \neq l_{1}$, and reciprocally.

The 14 asymmetric bijective transformation circular codes $\Pi_{\mathcal{A}}$ can also be partitioned into:

1. $N(4,3)=8$ symmetric bijective transformation circular codes $\Pi_{\mathcal{A}, 3}(X)$ at 3 letters.

$$
\begin{aligned}
\Pi_{\mathcal{A}, 3}(X)= & \left\{\pi_{10}(X):(A, C, G), \pi_{11}(X):(A, C, T), \pi_{12}(X):(A, G, C),\right. \\
& \pi_{13}(X):(A, G, T), \pi_{14}(X):(A, T, C), \pi_{15}(X):(A, T, G), \\
& \left.\pi_{16}(X):(C, G, T), \pi_{17}(X):(C, T, G)\right\}
\end{aligned}
$$

where $\pi_{i}(X):\left(l_{1}, l_{2}, l_{3}\right)$ is the $i$ th bijective transformation in the lexicographical order of the letter $l_{1} \in B$ into the letter $l_{2} \in B, l_{2} \neq l_{1}$, the letter $l_{2}$ into the letter $l_{3} \in B, l_{3} \neq l_{2} \neq l_{1}$, and the letter $l_{3}$ into the letter $l_{1}$.
2. $N(4,4)=6$ asymmetric bijective transformation circular codes $\Pi_{\mathcal{A}, 4}(X)$ at 4 letters

$$
\begin{aligned}
\Pi_{\mathcal{A}, 4}(X)= & \left\{\pi_{18}(X):(A, C, G, T), \pi_{19}(X):(A, C, T, G),\right. \\
& \pi_{20}(X):(A, G, C, T), \pi_{21}(X):(A, G, T, C), \\
& \left.\pi_{22}(X):(A, T, C, G), \pi_{23}(X):(A, T, G, C)\right\}
\end{aligned}
$$

where $\pi_{i}(X):\left(l_{1}, l_{2}, l_{3}, l_{4}\right)$ is the $i$ th bijective transformation in the lexicographical order of the letter $l_{1} \in B$ into the letter $l_{2} \in B, l_{2} \neq l_{1}$, the letter $l_{2}$ into the letter $l_{3} \in B, l_{3} \neq l_{2} \neq l_{1}$, and the letter $l_{3}$ into the letter $l_{4} \in B, l_{4} \neq l_{3} \neq l_{2} \neq l_{1}$, and the letter $l_{4}$ into the letter $l_{1}$.

Note that the transformation at $1\left(X=\pi_{0}(X)\right), 2,3$ and 4 letters are the transformations or order 1, 2, 3 and 4, respectively, according to the notation in (Fimmel, Danielli, and Strüngmann, 2013).

### 3.3.3.1.2 Complementary and non-complementary transformation

The 23 bijective complementary transformation circular codes $\Pi(X)$ of $X$ can also be partitioned into seven selt-complementary bijective transformation circular codes $\Pi_{\mathcal{C}}(X)=\left\{\pi_{3}(X), \pi_{4}(X), \pi_{7}(X), \pi_{8}(X), \pi_{9}(X), \pi_{1} 9(X), \pi_{2} 1(X)\right\}$ and 16 non selft-complementary bijective transformation circular codes $\Pi_{\overline{\mathcal{C}}}(X)=$ $\Pi(X) \backslash \Pi_{\mathcal{C}}(X)$ of $X$ (Table 3.3).

Table 3.3: The maximal $C^{3}$ self-complementary trinucleotide circular code $X=\pi_{0}(X)$ and its 23 bijective transformation circular codes $\Pi(X)=\left\{\pi_{1}(X), \ldots, \pi_{23}(X)\right\}$; the six symmetric bijective transformation circular codes $\Pi_{\mathcal{S}, 2}(X)=\left\{\pi_{1}(X), \ldots, \pi_{6}(X)\right\}$ at 2 letters, the three symmetric bijective transformation circular codes $\Pi_{\mathcal{S}, 2,2}(X)=\left\{\pi_{7}(X), \pi_{8}(X), \pi_{9}(X)\right\}$ of two disjoint transformations at 2 letters, the eight asymmetric bijective transformation circular codes $\Pi_{\mathcal{A}, 3}(X)=$ $\left\{\pi_{10}(X), \ldots, \pi_{17}(X)\right\}$ at 3 letters and the six asymmetric bijective transformation circular codes $\Pi_{\mathcal{S}, 4}(X)=\left\{\pi_{18}(X), \ldots, \pi_{23}(X)\right\}$ at 4 letters. The seven bijective transformations $\left\{\pi_{3}(X), \pi_{4}(X), \pi_{7}(X), \pi_{8}(X), \pi_{9}(X), \pi_{19}(X), \pi_{21}(X)\right\}$, in bold, are maximal $C^{3}$ self-complementary trinucleotide circular codes.

| $X=\pi_{0}(X)$ | AAC | AAT | ACC | ATC | ATT | CAG | CTC | CTG | GAA | GAC | GAG | GAT | GCC | GGC | GGT | GTA | GTC | GTT | TAC | TTC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\pi_{1}(X):(A, C)$ | CCA | CCT | CAA | CTA | CTT | ACG | ATA | ATG | GCC | GCA | GCG | GCT | GAA | GGA | GGT | GTC | GTA | GTT | TCA | TTA |
| $\pi_{2}(X):(A, G)$ | GGC | GGT | GCC | GTC | GTT | CGA | CTC | CTA | AGG | AGC | AGA | AGT | ACC | AAC | AAT | ATG | ATC | ATT | TGC | TTC |
| $\pi_{3}(X):(A, T)$ | TTC | TTA | TCC | TAC | TAA | CTG | CAC | CAG | GTT | GTC | GTG | GTA | GCC | GGC | GGA | GAT | GAC | GAA | ATC | AAC |
| $\pi_{4}(X):(C, G)$ | AAG | AAT | AGG | ATG | ATT | GAC | GTG | GTC | CAA | CAG | CAC | CAT | CGG | CCG | CCT | CTA | CTG | CTT | TAG | TTG |
| $\pi_{5}(X):(C, T)$ | AAT | AAC | ATT | ACT | ACC | TAG | TCT | TCG | GAA | GAT | GAG | GAC | GTT | GGT | GGC | GCA | GCT | GCC | CAT | CCT |
| $\pi_{6}(X):(G, T)$ | AAC | AAG | ACC | AGC | AGG | CAT | CGC | CGT | TAA | TAC | TAT | TAG | TCC | TTC | TTG | TGA | TGC | TGG | GAC | GGC |
| $\pi_{7}(\boldsymbol{X}):(A, C)(G, T)$ | CCA | CCG | CAA | CGA | CGG | ACT | AGA | AGT | TCC | TCA | TCT | TCG | TAA | TTA | TTG | TGC | TGA | TGG | GCA | GGA |
| $\pi_{8}(\boldsymbol{X}):(A, G)(C, T)$ | GGT | GGC | GTT | GCT | GCC | TGA | TCT | TCA | AGG | AGT | AGA | AGC | ATT | AAT | AAC | ACG | ACT | ACC | CGT | CCT |
| $\pi_{9}(X):(A, T)(C, G)$ | TTG | TTA | TGG | TAG | TAA | GTC | GAG | GAC | CTT | CTG | CTC | CTA | CGG | CCG | CCA | CAT | CAG | CAA | ATG | AAG |
| $\pi_{10}(X):(A, C, G)$ | CCG | CCT | CGG | CTG | CTT | GCA | GTG | GTA | ACC | ACG | ACA | ACT | AGG | AAG | AAT | ATC | ATG | ATT | TCG | TTG |
| $\pi_{11}(X):(A, C, T)$ | CCT | CCA | CTT | CAT | CAA | TCG | TAT | TAG | GCC | GCT | GCG | GCA | GTT | GGT | GGA | GAC | GAT | GAA | ACT | AAT |
| $\pi_{12}(X):(A, G, C)$ | GGA | GGT | GAA | GTA | GTT | AGC | ATA | ATC | CGG | CGA | CGC | CGT | CAA | CCA | CCT | CTG | CTA | CTT | TGA | TTA |
| $\pi_{13}(X):(A, G, T)$ | GGC | GGA | GCC | GAC | GAA | CGT | CAC | CAT | TGG | TGC | TGT | TGA | TCC | TTC | TTA | TAG | TAC | TAA | AGC | AAC |
| $\pi_{14}(X):(A, T, C)$ | TTA | TTC | TAA | TCA | TCC | ATG | ACA | ACG | GTT | GTA | GTG | GTC | GAA | GGA | GGC | GCT | GCA | GCC | CTA | CCA |
| $\pi_{15}(X):(A, T, G)$ | TTC | TTG | TCC | TGC | TGG | CTA | CGC | CGA | ATT | ATC | ATA | ATG | ACC | AAC | AAG | AGT | AGC | AGG | GTC | GGC |
| $\pi_{16}(X):(C, G, T)$ | AAG | AAC | AGG | ACG | ACC | GAT | GCG | GCT | TAA | TAG | TAT | TAC | TGG | TTG | TTC | TCA | TCG | TCC | CAG | CCG |
| $\pi_{17}(X):(C, T, G)$ | AAT | AAG | ATT | AGT | AGG | TAC | TGT | TGC | CAA | CAT | CAC | CAG | CTT | CCT | CCG | CGA | CGT | CGG | GAT | GGT |
| $\pi_{18}(X):(A, C, G, T)$ | CCG | CCA | CGG | CAG | CAA | GCT | GAG | GAT | TCC | TCG | TCT | TCA | TGG | TTG | TTA | TAC | TAG | TAA | ACG | AAG |
| $\pi_{19}(X):(A, C, T, G)$ | CCT | CCG | CTT | CGT | CGG | TCA | TGT | TGA | ACC | ACT | ACA | ACG | ATT | AAT | AAG | AGC | AGT | AGG | GCT | GGT |
| $\pi_{20}(X):(A, G, C, T)$ | GGT | GGA | GTT | GAT | GAA | TGC | TAT | TAC | CGG | CGT | CGC | CGA | CTT | CCT | CCA | CAG | CAT | CAA | AGT | AAT |
| $\pi_{21}(X):(A, G, T, C)$ | GGA | GGC | GAA | GCA | GCC | AGT | ACA | ACT | TGG | TGA | TGT | TGC | TAA | TTA | TTC | TCG | TCA | TCC | CGA | CCA |
| $\pi_{22}(X):(A, T, C, G)$ | TTG | TTC | TGG | TCG | TCC | GTA | GCG | GCA | ATT | ATG | ATA | ATC | AGG | AAG | AAC | ACT | ACG | ACC | CTG | CCG |
| $\pi_{23}(X):(A, T, G, C)$ | TTA | TTG | TAA | TGA | TGG | ATC | AGA | AGC | CTT | CTA | CTC | CTG | CAA | CCA | CCG | CGT | CGA | CGG | GTA | GGA |

### 3.3.3.2 MAIN PROPERTIES OF THE 23 BIJECTIVE TRANSFORMATION CIRCULAR CODES

Proposition 3.1. The 23 bijective transformation circular codes $\Pi(X)$ of $X$ are $C^{3}$.

Proof. By letter invariance, $\Pi(X)$ belongs to the set of the $221,328 C^{3}$ trinucleotides circular codes or by Proposition 3 in Michel and Pirillo, 2010 or by Theorem 1 in Fimmel et al., 2014.

Proposition 3.2. The seven bijective transformation circular codes $\Pi_{\mathcal{C}}(X)=\left\{\pi_{3}(X), \pi_{4}(X), \pi_{7}(X), \pi_{8}(X), \pi_{3}(X), \pi_{9}(X), \pi_{1} 9(X), \pi_{2} 1(X)\right\}$ are $C^{3}$ self-complementary.

Proof. By letter invariance for the complementarity map $\mathcal{C}, \Pi_{\mathcal{C}}(X)$ belongs to the set of the $216 C^{3}$ self-complementary trinucleotides circular codes (Arquès and Michel, 1996) or by Proposition 3 in Michel and Pirillo, 2010 or by Theorem 2 in Fimmel et al., 2014.

Proposition 3.3. The probability PrRFC (Definition 2.21 in Michel, 2014) of reading frame coding (RFC) of the 23 bijective transformation circular codes $\Pi(X)$ of $X$ are obviously all equal to the probability $\operatorname{Pr} R F C=81.3 \%$ of $X$ (Section 2.2.2.(vi) in Michel, 2014).

### 3.3.4 Statistical analysis of $X$ circular code motifs

Several minor methods were developed to aid us interpret and compare our data to uncover significance that would be otherwise difficult to notice to naked eye.

### 3.3.4.1 Coverage of $X$ motifs in tRNA

The following method was used to estimate the probability of a base belonging to a circular code motif across several sequences. This is ideally used for sequences of similar lengths, i.e. a set of tRNA or $16 \mathrm{~S} / 18 \mathrm{~S}$ rRNA sequences. This approach will gather the motifs from each sequence and calculate the probability for a base in a sequence to belong to a circular code motif.

Let $m$ be a set of $X$ motifs, and their respective start and end positions be the set $P=\left\{\left[b_{1} . . e_{1}\right], \ldots,\left[b_{m} . . e_{m}\right]\right\}$ in a nucleic acid region designated $R=[a . . b]$. Then the $\operatorname{Interval}(P)=\left\{\left[\min _{1} . . \max _{1}\right], \ldots,\left[\min _{n} . . \max _{n}\right]\right\}$ is the union of the ranges $b_{1}$ to $e_{1}, \ldots, b_{m}$ to $e_{m}$.

Therefore, the Coverage $(X, R)$ giving the probability of sites in nucleic acid region $R$ occupied by $X$ motifs is:

$$
\begin{equation*}
\operatorname{Coverage}(X, R)=\frac{1}{b-a+1} \sum_{i=1}^{n}\left(\max _{i}-\min _{i}+1\right) \tag{3.5}
\end{equation*}
$$

### 3.3.4.2 Expectation of the occurrence number of a motif

The expectation $E\left[N\left(m_{c h r}(X)\right)\right]$ of the occurrence number $N\left(m_{c h r}(X)\right)$ of an $X$ motif $m_{X}$ in chromosome $c h r$ of a genome can easily be calculated with Bernoulli model:

$$
\begin{equation*}
E\left[N\left(m_{c h r}(X)\right)\right]=\left(N_{c h r}-3 l+1\right)\left(\frac{20}{64}\right)^{l} . \tag{3.6}
\end{equation*}
$$

where $N_{c h r}$ is the total number of bases (size) of a chromosome chr, l= $l\left(m_{c h r}(X)\right)$ is the length of the motif in trinucleotides and the term $\frac{20}{64}$ is the probability of occurrence of a trinucleotide $X$ (we recall $X$ is maximal, thus it has 20 trinucleotides). Given that any $X$ motif of length greater than four trinucleotides cannot overlap by definition of circular code.

### 3.3.4.3 Ratio of $X$ motifs in Coding and non-CODing Regions

The statistical analysis of $X$ motifs $m(X)$ in a genome is based on two simple ratios: a base ratio of coding/non-coding for characterizing the base proportion of coding regions in a genome and a base ratio of $X$ motifs in coding/non-coding for analysing the base proportion of $X$ motifs $m(X)$ in coding and non-coding regions of a genome.

The base ratio $r(\mathcal{G})$ of coding/non-coding in a genome $\mathcal{G}$ is defined as follows:

$$
\begin{equation*}
r(\mathcal{G})=\frac{N\left(\mathcal{G}_{C}\right)}{N\left(\mathcal{G}_{\bar{C}}\right)} \tag{3.7}
\end{equation*}
$$

where $N\left(\mathcal{G}_{C}\right)$ is the total base number in coding regions in a given genome $\mathcal{G}$ and $N\left(\mathcal{G}_{\bar{C}}\right)$ is the total base number in non-coding regions in $\mathcal{G}$.

Remark 3.3. $N\left(\mathcal{G}_{C}\right)+N\left(\mathcal{G}_{\bar{C}}\right)=N(\mathcal{G})$ where $N(\mathcal{G})$ is the total base number (size) of a genome $\mathcal{G}$.

Remark 3.4. When $r(\mathcal{G})<1$, the total base number $N(\mathcal{G})$ of all coding regions
$\mathcal{G}_{C}$ in a genome $\mathcal{G}$ is less than the total base number $N\left(\mathcal{G}_{\bar{C}}\right)$ of all non-coding regions $\mathcal{G}_{\bar{C}}$ in $\mathcal{G}$, and conversly when $r(\mathcal{G})>1$.

Example 3.1. With the genome $\mathcal{G}=$ Anolis carolinensis, $N\left(\mathcal{G}_{C}\right)=16670366$ and $N\left(\mathcal{G}_{\bar{C}}\right)=1064974225$ (see Appendix A), then $r(\mathcal{G})=1.6 \%$.

In order to study a greater variety of $X$ motifs, i.e. not necessarily large, the two constraints on length and cardinality (composition) defined in Equation 3.4 are relaxed. Thus, the $X$ motifs studied in this section are based on the following conditions

$$
\left\{\begin{array}{l}
n=l(m(Y)) \geq 10 \text { trinucleotides }  \tag{3.8}\\
\operatorname{Card}(\{W(m(Y))\}) \geq 5 \text { trinucleotides }
\end{array}\right.
$$

The base ratio $r_{m(X)}(\mathcal{G})$ of $X$ motifs in coding/non-coding regions in a genome $\mathcal{G}$ is defined as follows:

$$
\begin{equation*}
r_{m(X)}(\mathcal{G})=\frac{P\left(m_{\mathcal{G}_{C}}(X)\right)}{P\left(m_{\mathcal{G}_{\mathscr{C}}}(X)\right)} \tag{3.9}
\end{equation*}
$$

where the probability $P\left(m_{\mathcal{G}_{C}}(X)\right)=\frac{N\left(m_{\mathcal{G}_{C}}(X)\right)}{N\left(\mathcal{G}_{C}\right)}$ is the total base number $N\left(m_{\mathcal{G}_{C}}(X)\right)$ of $X$ motifs in the coding regions of a genome $\mathcal{G}$ divided by the total base number $N\left(\mathcal{G}_{C}\right)$ of coding regions in $m \mathcal{G}$ (see Equation 3.7), and the probability $P\left(m_{\mathcal{G}_{\bar{C}}}(X)\right)=\frac{N\left(m_{\mathcal{G}_{C}}(X)\right)}{N\left(\mathcal{G}_{\bar{C}}\right)}$ is the total base number $N\left(m_{\mathcal{G}_{\bar{C}}}(X)\right)$ of $X$ motifs in the non-coding regions of $\mathcal{G}$ divided by the total base number $N\left(\mathcal{G}_{\bar{C}}\right)$ of non-coding regions in $\mathcal{G}$ (see Equation 3.7).

Remark 3.5. A ratio $r_{m(X)}(\mathcal{G})=1$ means that the proportion of $X$ motifs in coding and non coding regions is identical in the genome. A ratio $r_{m(X)}(\mathcal{G})<1$ means that there is preferential occurrence of $X$ motifs in non-coding regions of $\mathcal{G}$. Conversely, a $r_{m(X)}(\mathcal{G})>1$ means that there is a preferential occurrence of $X$ motifs in coding regions of $\mathcal{G}$.

The following algorithm computes the numbers $N\left(m_{\mathcal{G}_{C}}(X)\right)$ and $N\left(m_{\mathcal{G}_{\widetilde{C}}}(X)\right)$ of $X$ motifs in coding and non-coding regions, respectively, of a genome $\mathcal{G}$. The sequence from genome $\mathcal{G}$ is laid with two markers, the first marker labels the nucleotides that belong to an $X$ motif and the second marker labels the nucleotides that are in a coding region. The number $N\left(m_{\mathcal{G}_{\bar{C}}}(X)\right)$ of $X$ motifs in non-coding regions is the number of nucleotides that have only the first marker. Note that
if an $X$ motif overlaps a coding and non-coding region, its nucleotides are split accordingly.

### 3.4 Ribosome study tools

### 3.4.1 Multiple sequence alignment

The rRNA sequences from the collected PDB entries were aligned using a multiple sequence alignment (MSA) software, CLUSTAL X 2.0.12 (Chenna et al., 2003; Higgins, Thompson, and Gibson, 1996; Jeanmougin et al., 1998; Larkin et al., 2007; Thompson et al., 1997), to help use better visualize similarities between the sequences of the various organisms at our disposal. The alignments were done on four groups. Group one was the bacterial group, which aligned the rRNA of E. coli and T. thermophilus. The second group was prokaryotic which included the rRNA of E. coli, T. thermophilus and P. furiosus. Third group was the eukaryotic, which included $S$. cerevisiae, T. aestivum and H. sapiens $(S$. aleracea, chloroplast, was left out due to its size difference with other eukaryotic rRNAs).

| 3J5T:A | CGTTAAGTCGACCGCCTGGGGAGTACGGCCGCAAGGTTAAAACTCAAATGAATTGACGGG |
| :---: | :---: |
| 3I8G:A | CGTTAAGCGCGCCGCCTGGGGAGTACGGCCGCAAGGCTGAAACTCAAAGGAATTGACGGG |
| 3J20:2 | CGTTAAGCCCGCCGCCTGGGGAGTACGGCCGCAAGGCTGAAACTTAAAGGAATTGGCGGG |

Figure 3.1: The output of a multiple sequence alignment using Clustal $X$, where the conventional asterisk (*) designates a conserved nucleotide.

The normal output of a MSA (Figure 3.1) was modified to highlight $X$ circular code motifs found in the sequence with an alternation between green and red for each motif, e.g. in Figure 3.2 (b) we see the switch of colors between two consecutive motifs. The color blue (Figure 3.1 (a)) was used to highlight shared nucleotides between two overlapping motifs (due to length being less than 12 or different frames). Finally, the asterisk (*) in the modified output will designated a nucleotide base that belongs to a circular code motif in all the aligned sequences.

### 3.4.2 Molecule viewer

The reason behind conducting a study on PDB entries is the ability to visualize the results, which is made possible using a molecule viewer. Jmol is an opensource Java viewer (Hanson, 2010; Herráez, 2006; Willighagen, 2001) for chemical structures in 3D (www.jmol.org). It allows the reading of a variety of file formats


Figure 3.2: The modified output of the multiple sequence alignment. The asterisk (*) designates a nucleotide belonging to $X$ circular code motif in all sequences. Coloured nucleotides are $X$ circular code motifs.
and high-performance 3D rendering with no hardware requirements. This allows us to examine the spatial location a motif occupies and its proximity to important or interesting regions of the ribosome.

Our application allows us to generate Jmol scripts for any PDB entry while highlighting the $X$ circular code motifs found. Our scripts will hide protein sequences since they do not interest us in this study. It will give different structural shapes and colors to the rRNA, tRNA and mRNA sequences to better differentiate them.


Figure 3.3: The window of Jmol with an unscripted and unformatted PDB entry of the Homo sapiens ribosome on display (PDB ID: 3J3D).

### 3.5 Repeated motifs

The unitary circular code motifs ( $U C C$ motifs or repeated motifs) are generated from the unitary circular codes ( $U C C$ defined in 2.6). They are defined by two parameters: their equivalence classes and their length in nucleotides.

### 3.5.1 Dinucleotide unitary circular code motifs

A repeated dinucleotide $d i^{+}=\left(l_{1} l_{2}\right)^{+}$with $l_{1} l_{2} \in B$ and $l_{1} \neq l_{2}$ belongs to one of the $\frac{16-4}{2}=6$ equivalence classes $\left\{\left(l_{1} l_{2}\right)^{+},\left(l_{2} l_{1}\right)^{+}\right\}$(Definition 2.13): $\left\{(A C)^{+},(C A)^{+}\right\},\left\{(A G)^{+},(G A)^{+}\right\},\left\{(A T)^{+},(T A)^{+}\right\},\left\{(C G)^{+},(G C)^{+}\right\}$, $\left\{(C T)^{+},(T C)^{+}\right\}$and $\left\{(G T)^{+},(T G)^{+}\right\}$. By convention, the dinucleotide UCC motifs are defined by the six repeated dinucleotide which are the 1st repeated motif in lexicographical order in each equivalence class:

$$
\begin{equation*}
D i=\left\{(A C)^{+},(A G)^{+},(A T)^{+},(C G)^{+},(C T)^{+},(G T)^{+}\right\} \tag{3.10}
\end{equation*}
$$

The repeated dinucleotides $d i^{n}$ studied have length $l=n \times|d i| \geq 30$ (|di| being the number of letters of $d i$ ), i.e. $n \geq 15$.

### 3.5.2 TRINUCLEOTIDE UNITARY CIRCULAR CODE MOTIFS

A repeated trinucleotide tri ${ }^{+}=\left(l_{1} l_{2} l_{3}\right)$ with $l_{1} l_{2} l_{3} \in B$ and $l_{1} l_{2} \neq l_{2} l_{3}$ belongs to one of the $\frac{64-4}{3}=20$ equivalence classes $\left\{\left(l_{1} l_{2} l_{3}\right)^{+},\left(l_{2} l_{3} l_{1}\right)^{+},\left(l_{3} l_{1} l_{2}\right)^{+}\right\}$(Definition 2.13): $\left\{(A A C)^{+},(A C A)^{+},(C A A)^{+}\right\}, \ldots,\left\{(G T T)^{+},(T T G)^{+},(T G T)^{+}\right\}$. By convention and similarly as Di motifs, the trinucleotide UCC motifs, Tri, are defined by:

$$
\begin{align*}
T r i^{+}=\{ & (A A C)^{+},(A A G)^{+},(A A T)^{+},(A C C)^{+},(A C G)^{+},(A C T)^{+},(A G C)^{+}, \\
& (A G G)^{+},(A G T)^{+},(A T C)^{+},(A T G)^{+},(A T T)^{+},(C C G)^{+},(C C T)^{+}, \\
& \left.(C G G)^{+},(C G T)^{+},(C T G)^{+},(C T T)^{+},(G G T)^{+},(G T T)^{+}\right\} . \tag{3.11}
\end{align*}
$$

The repeated trinucleotides $\operatorname{tri}^{n}$ studied have length $l=n \times \mid$ tri $\mid \geq 30$ nucleotides, i.e. $n \geq 10$.

### 3.5.3 Tetranucleotide unitary circular code motifs

A repeated tetranucleotide tetra ${ }^{+}=\left(l_{1} l_{2} l_{3} l_{4}\right)$ with $l_{1}, l_{2}, l_{3}, l_{4} \in B$ and $l_{1} l_{2} \neq l_{3} l_{4}$ belongs to one of the $\frac{256-16}{4}=60$ equivalence classes $\left\{\left(l_{1} l_{2} l_{3} l_{4}\right)^{+},\left(l_{2} l_{3} l_{4} l_{1}\right)^{+},\left(l_{3} l_{4} l_{1} l_{2}\right)^{+},\left(l_{4} l_{1} l_{2} l_{3}\right)^{+}\right\}$(Definition 2.13). By convention and similarly as $D i$ motifs, the tetranucleotide $U C C$ motifs, Tetra, are defined by:

$$
\begin{equation*}
\text { Tetra }=\left\{(A A A C)^{+}, \ldots,(G T T T)^{+}\right\} \tag{3.12}
\end{equation*}
$$

The repeated tetranucleotides tetra $^{n}$ studied have length $l=n \times \mid$ tetra $\mid \geq 28$ nucleotides, i.e. $n \geq 7$.

### 3.5.4 Statistical analysis of repeated motifs

### 3.5.4.1 Occurrence number of unitary circular code motifs

Let $r^{n} \in\left\{d i^{n}\right.$, tri $^{n}$, tetra $\left.{ }^{n}\right\}$ be a repeated motif $r$ of nucleotide length $n \times|r|$ (with $|r| \in\{2,3,4\}$ being the number of letters in $r$ ) where $r=d i$ for repeated dinucleotide $d \imath^{n}, r=t r i$ for a repeated trinucleotide trin ${ }^{n}$ and $r=$ tetra for a repeated tetranucleotide tetra ${ }^{n}$. The number $N\left(r^{n}, \mathcal{G}\right)$ counts the occurrences of a repeated motif $r^{n}$ for a given number $n$ in a eukaryotic genome $\mathcal{G}$. Then, the occurrence number $N\left(r^{+}\right)$or a repeated motif $r^{+} \in\left\{d i^{+}\right.$, tri $i^{+}$,tetra $\left.{ }^{+}\right\}$in the genomes of eukaryotes $\mathbb{E}$ is obtained by summing for all genomes $\mathcal{G}$ in $\mathbb{E}$ and for all $n$

$$
\begin{equation*}
N\left(r^{+}\right)=\sum_{\mathcal{G} \in \mathbb{E}} \sum_{n} N\left(r^{n}, \mathcal{G}\right) \tag{3.13}
\end{equation*}
$$

With $n$ varying between the different UCC, being $n \geq 15$ for computing $N\left(d \imath^{+}\right)$ of a repeated dinucleotide $d i^{+}, n \geq 10$ for computing $N\left(t r i^{+}\right)$of a repeated trinucleotide $\operatorname{tri}^{+}$and $n \geq 7$ for computing $N\left(\right.$ tetra $\left.^{+}\right)$of a repeated tetranucleotide tetra ${ }^{+}$. These occurrence numbers $N\left(d i^{+}\right), N\left(t r i^{+}\right)$and $N\left(t e t r a^{+}\right)$are computed in the genomes of eukaryotes using the following algorithm.

The algorithm searches for repeated motifs in a DNA sequence such that their lengths are greater than or equal to the parameter minsize and returns a frequency map for the association of a word and how many times it was successively repeated. The algorithm is generic in regards to the input set as a parameter. It determines the number of frames required with respect to word

```
Algorithm 3.3 The algorithm RepeatsFinder gives the occurrence numbers of
repeats in any sequence.
```

```
Read sequence
```

Read sequence
INIT Y AS a set of words
INIT Y AS a set of words
INIT minsize AS the minimum number of words in a repeats motif
INIT minsize AS the minimum number of words in a repeats motif
INIT wordsize from Y
INIT wordsize from Y
INIT mapFreq AS a map using the association of a word and number
INIT mapFreq AS a map using the association of a word and number
of repeats as key with their frequency as value
of repeats as key with their frequency as value
FOR EACH frame in wordsize
FOR EACH frame in wordsize
INIT wordCurrent AS empty
INIT wordCurrent AS empty
INIT streak AS 0, number of successive wordCurrent
INIT streak AS 0, number of successive wordCurrent
FOR EACH word in sequence starting from frame AS wordSeq
FOR EACH word in sequence starting from frame AS wordSeq
IF Y contains wordSeq THEN
IF Y contains wordSeq THEN
IF wordCurrent equals wordSeq THEN
IF wordCurrent equals wordSeq THEN
increment streak by 1
increment streak by 1
ELSE
ELSE
IF streak is greater than or equal to minsize THEN
IF streak is greater than or equal to minsize THEN
increment the frequency of wordCurrent with size equal
increment the frequency of wordCurrent with size equal
to streak in mapFreq by 1
to streak in mapFreq by 1
ENDIF
ENDIF
INIT wordCurrent As wordSeq
INIT wordCurrent As wordSeq
INIT streak AS 1
INIT streak AS 1
ENDIF
ENDIF
ELSE
ELSE
IF streak is greater than or equal to minsize THEN
IF streak is greater than or equal to minsize THEN
increment the frequency of wordCurrent with streak in
increment the frequency of wordCurrent with streak in
mapFreq by 1
mapFreq by 1
ENDIF
ENDIF
INIT wordCurrent As empty
INIT wordCurrent As empty
INIT streak AS 0
INIT streak AS 0
ENDIF
ENDIF
ENDFOR
ENDFOR
ENDFOR

```
        ENDFOR
```

length (two frames for dinucleotides, three frames for trinucleotides and four frames for tetranucleotides). This approach allows us to retrieve all the repeated motifs without the issue of overlaps between different frames because of the nature of unitary circular code.

Example 3.2. If the trinucleotide tri $=A A C$ occurs with two repeats $t r i^{n_{1}}$ with $n_{1}=10$ in a genome $\mathcal{G}_{1}$, i.e. $N\left(\operatorname{tri}^{10}, \mathcal{G}_{1}\right)=2$, and three repeats $\operatorname{tri}^{n_{2}}$ with $n_{2}=20$ in a genome $\mathcal{G}_{2}$, i.e. $N\left(t r i^{20}, \mathcal{G}_{2}\right)=3$, the occurrence number $N\left(t r i^{+}\right)$of the repeated trinucleotide $(A A C)^{+}$in the genomes of eukaryotes $\mathbb{E}$ is equal to $N\left(\right.$ tri $\left.^{+}\right)=N\left(t^{10}, \mathcal{G}_{1}\right)+N\left(\right.$ tri $\left.^{20}, \mathcal{G}_{2}\right)=2+3=5($ Equation 3.13) .

### 3.5.4.2 BaSE NUMBER OF UNITARY CIRCULAR CODE MOTIFS

The base number $B\left(r^{+}\right)$of a repeated motif $r^{+} \in\left\{d i^{+}\right.$, tri $i^{+}$,tetra $\left.{ }^{+}\right\}$in the genomes of eukaryotes $\mathbb{E}$ is:

$$
\begin{equation*}
B\left(r^{+}\right)=|r| \sum_{\mathcal{G} \in \mathbb{E}} \sum_{n} N\left(r^{n}, \mathcal{G}\right) \times n \tag{3.14}
\end{equation*}
$$

where $N\left(r^{n}, \mathcal{G}\right)$ is defined in Section 3.5.4.1 and with $n \geq 15$ for computing $B\left(d i^{+}\right)$of a repeated dinucleotide $d i^{+}, n \geq 10$ for computing $B\left(t r i^{+}\right)$of a repeated trinucleotide tri ${ }^{+}$and $n \geq 7$ for computing $B\left(t e t r a^{+}\right)$of a repeated tetranucleotide tetra+,$|r| \in\{2,3,4\}$ being the number of letters of $r$.

Example 3.3. If the trinucleotide tri $=A A C$ occurs with two repeats tri ${ }^{n_{1}}$ with $n_{1}=10$ in a genome $\mathcal{G}_{1}$, i.e. $N\left(r^{n_{1}}, \mathcal{G}_{1}\right)=2$, and three repeats $t^{n_{2}}$ with $n_{2}=20$ in a genome $\mathcal{G}_{2}$, i.e. $N\left(r^{n_{2}}, \mathcal{G}_{2}\right)=3$, then the base number $B\left(\right.$ tri $\left.^{+}\right)$ of the repeated trinucleotide $A A C^{+}$in the genomes of eukaryotes $\mathbb{E}$ is equal to $B\left(\right.$ tri $\left.^{+}\right)=|\operatorname{tri}|\left(N\left(r^{n_{1}}, \mathcal{G}_{1}\right) \times n_{1}+N\left(r^{n_{2}}, \mathcal{G}_{2}\right) \times n_{2}\right)=3(2 \times 10+3 \times 20)=240$ (Equation 3.14).

### 3.5.4.3 Total base number of unitary circular code motifs

The total base number $B\left(R^{+}, \mathcal{G}\right)$ of all repeated motifs $R^{+} \in\left\{D i^{+}\right.$, Tri $^{+}$, Tetra $\left.{ }^{+}\right\}$ (Equations 3.10, 3.11 and 3.12)in a genome is:

$$
\begin{equation*}
B\left(R^{+}, \mathcal{G}\right)=|r| \sum_{r+\in R^{+}} \sum_{n} N\left(r^{n}, \mathcal{G}\right) \times n \tag{3.15}
\end{equation*}
$$

where $N\left(r^{n}, \mathcal{G}\right)$ is defined in Section 3.5.4.1 and with $n \geq 15$ for computing $B\left(D i^{+}, \mathcal{G}\right)$ of all repeated dinucleotide $d i^{+} \in D i^{+}$(Equation 3.10), $n \geq 10$ for computing $B\left(R i^{+}, \mathcal{G}\right)$ of all repeated trinucleotide ri $^{+} \in \operatorname{Tri}^{+}$(Equation 3.11) and $n \geq 7$ for computing $B\left(\right.$ Tetra $\left.^{+}, \mathcal{G}\right)$ of all repeated tetranucleotide tetra ${ }^{+} \in$ Tetra $^{+}$(Equation 3.12), $|r| \in\{2,3,4\}$ being the number of letters of $r$.

Example 3.4. If the trinucleotide $t r i_{1}=A A C$ occurs with two repeats $t r l_{1}^{n_{1}}$ with $n_{1}=10$ in a genome $\mathcal{G}$, i.e. $N\left((A A C)^{n_{1}}, \mathcal{G}\right)=2$, and three repeats $t_{1}^{n_{2}}$ with $n_{2}=20$ in a genome $\mathcal{G}_{2}$, i.e. $N\left((A A C)^{n_{2}}, \mathcal{G}\right)=3$, and if the trinucleotide
 $\mathcal{G}$, i.e. $N\left((A A G)^{n_{3}}, \mathcal{G}\right)=3$, then the total base number $B\left(T r i^{+}, \mathcal{G}\right)$ of repeated motifs $\mathrm{Tri}^{+}$in the genome $\mathcal{G}$ is equal to $B\left(T r i^{+}, \mathcal{G}\right)=|\operatorname{tri}|\left(N\left((A A C)^{n_{1}}, \mathcal{G}\right) \times n_{1}+\right.$
$\left.N\left((A A C)^{n_{2}}, \mathcal{G}\right) \times n_{2}+N\left((A A G)^{n_{3}}, \mathcal{G}\right) \times n_{3}\right)=3(2 \times 10+3 \times 20+4 \times 30)=600$ (Equation 3.15).

In order to normalize the total number $B\left(R^{+}, \mathcal{G}\right)$ (Equation 3.15) for eukaryotic genomes of different sizes, the ratio $r\left(R^{+}, \mathcal{G}\right)$ gives the proportion of the total base number $B\left(R^{+}, \mathcal{G}\right)$ of all repeated motifs $R^{+}$in a eukaryotic genomes $\mathcal{G}$ of size $N(\mathcal{G})$ (Table 3.2) is defined by

$$
\begin{equation*}
r\left(R^{+}, \mathcal{G}\right)=\frac{B\left(R^{+}, \mathcal{G}\right)}{N(\mathcal{G})} \tag{3.16}
\end{equation*}
$$

Finally, $\bar{r}\left(R^{+}\right)$is the mean of the ratios $r\left(R^{+}, \mathcal{G}\right)$ in the genomes of eukaryotes $\mathbb{E}$

$$
\begin{equation*}
\bar{r}\left(R^{+}\right)=\frac{1}{|\mathbb{E}|} \prod_{\mathcal{G} \in \mathbb{E}} r\left(R^{+}, \mathcal{G}\right) \tag{3.17}
\end{equation*}
$$

where $|\mathbb{E}|$ is the number of genomes in $\mathbb{E}$ and $\tilde{r}\left(R^{+}\right)$is the median of the ratios $r\left(R^{+}, \mathcal{G}\right)$ in the genomes of eukaryotes $\mathbb{E}$.

### 3.6 OCCURRENCE NUMBER OF TRINUCLEOTIDE PAIRS

In order to identify a new property of the circular code $X$, we study the occurrence of the two consecutive trinucleotides $t_{1} t_{2} \in B^{6}$, called trinucleotide pairs, where $t_{1}, t_{2} \in B^{3}\left(\left|B^{6}\right|=4096 t_{1} t_{2}\right.$ motifs) in the eukaryotic gene sequences. The trinucleotide pairs $t_{1} t_{2} \in X^{2}$ where $t_{1}, t_{2} \in X\left(|X|^{2}=400 t_{1} t_{2}\right.$ motifs $)$ are associated to the circular $X$.

The number $N\left(t_{1} t_{2}, \mathcal{G}\right)$ counts the occurrences of a trinucleotide pair $t_{1} t_{2} \in B^{6}$ in (all) the gene sequences of a eukaryotic genome $\mathcal{G}$. Note that the number $N\left(t_{1} t_{2}, \mathcal{G}\right)=N\left(t^{n}, \mathcal{G}\right)$ is a repeated trinucleotide $t^{n}$ where $n=2$ and of nucleotide length $n \times|t|=2=6$ (Section 3.5.4.1). Then, the occurrence number $N\left(t_{1} t_{2}\right)$ of a trinucleotide pair $t_{1} t_{2} \in B^{6}$ in the gene sequences of eukaryotes $\mathbb{E}$ is

$$
\begin{equation*}
N\left(t_{1} t_{2}\right)=\sum_{\mathcal{G} \in \mathbb{E}} N\left(t_{1} t_{2}, \mathcal{G}\right) \tag{3.18}
\end{equation*}
$$

The observed probability $P\left(t_{1} t_{2}\right)$ of a trinucleotide pair $t_{1} t_{2} \in B^{6}$ in the gene sequences of $\mathbb{E}$ is

$$
\begin{equation*}
P\left(t_{1} t_{2}\right)=\frac{N\left(t_{1} t_{2}\right)}{\sum_{t_{1} t_{2} \in B^{6}} N\left(t_{1} t_{2}\right)} . \tag{3.19}
\end{equation*}
$$

Due to the codon usage, in particular, this probability $P\left(t_{1} t_{2}\right)$ must be normalized. The observed probability $P(t)$ of a trinucleotide $t \in B^{3}$ in the gene sequences of $\mathbb{E}$ is

$$
\begin{equation*}
P(t)=\frac{N(t)}{\sum_{t \in B^{3}} N(t)} \tag{3.20}
\end{equation*}
$$

with $N(t)=\sum_{\mathcal{G} \in \mathbb{E}} N(t, \mathcal{G})$ where $N(t, \mathcal{G})$ (a repeated trinucleotide $t^{n}$ where $n=1$ ) is the occurrence number of $t$ in the gene sequences of a eukaryotic genome $\mathcal{G}$. By taking the hypothesis of independent events then the estimated theoretical probability $\hat{P}\left(t_{1} t_{2}\right)$ of a trinucleotide pair $t_{1} t_{2} \in B^{6}$ in the gene sequences of $\mathbb{E}$ is

$$
\begin{equation*}
\hat{P}\left(t_{1} t_{2}\right)=P\left(t_{1}\right) \times P\left(t_{2}\right) \tag{3.21}
\end{equation*}
$$

Therefore, the observed/theoretical ratio $r\left(t_{1} t_{2}\right)$ of trinucleotide pair $t_{1} t_{2} \in$ $B^{6}$ in the genes of $\mathbb{E}$ is equal to

$$
\begin{equation*}
r\left(t_{1} t_{2}\right)=\frac{P\left(t_{1} t_{2}\right)}{\hat{P}\left(t_{1} t_{2}\right)} \tag{3.22}
\end{equation*}
$$

Two other ratios also analyse the occurrence of trinucleotide pairs in the eukaryotic gene sequences. The observed probability $P\left(t_{1} t_{2}, \mathcal{G}\right)$ of a trinucleotide pair $t_{1} t_{2} \in B^{6}$ in the gene sequences of a eukaryotic genome $\mathcal{G}$ is

$$
\begin{equation*}
P\left(t_{1} t_{2}, \mathcal{G}\right)=\frac{N\left(t_{1} t_{2}, \mathcal{G}\right)}{\sum_{t_{1} t_{2} \in B^{6}} N\left(t_{1} t_{2}, \mathcal{G}\right)} \tag{3.23}
\end{equation*}
$$

Equation 3.23 for the gene sequences of a eukaryotic genome $\mathcal{G}$ is similar to Equation 3.19 for the gene sequences of eukaryotic $\mathbb{E}$. Similarly as previously, the observed probability $P(t, \mathcal{G})$ of a trinucleotide $t \in B^{3}$ in the gene sequences of genome $\mathcal{G}$ is

$$
\begin{equation*}
P(t, \mathcal{G})=\frac{N(t, \mathcal{G})}{\sum_{t \in B^{3}} N(t, \mathcal{G})} \tag{3.24}
\end{equation*}
$$

where $N(t, \mathcal{G})$ defined in Equation 3.20 is the occurrence number of $t$ in the gene
sequences of genome $\mathcal{G}$. By taking the hypothesis of independent events then theoretical probability $\hat{P}\left(t_{1} t_{2}, \mathcal{G}\right)$ of a trinucleotide pair $t_{1} t_{2} \in B^{6}$ in the gene sequences of genome $\mathcal{G}$ is:

$$
\begin{equation*}
\hat{P}\left(t_{1} t_{2}, \mathcal{G}\right)=P\left(t_{1}, \mathcal{G}\right) \times P\left(t_{2}, \mathcal{G}\right) \tag{3.25}
\end{equation*}
$$

Then, the observed/theoretical ratio $r\left(t_{1} t_{2}, \mathcal{G}\right)$ of a trinucleotide pair $t_{1} t_{2} \in B^{6}$ in the gene sequences of $\mathcal{G}$ is equal to:

$$
\begin{equation*}
r\left(t_{1} t_{2}, \mathcal{G}\right)=\frac{P\left(t_{1} t_{2}, \mathcal{G}\right)}{\hat{P}\left(t_{1} t_{2}, \mathcal{G}\right)} \tag{3.26}
\end{equation*}
$$

Finally, $\bar{r}\left(t_{1} t_{2}\right)$ is the mean of the observed/theoretical ratios $r\left(t_{1} t_{2}, \mathcal{G}\right)$ of a trinucleotide pair $t_{1} t_{2} \in B^{6}$ in the gene sequences of eukaryotic $\mathbb{E}$

$$
\begin{equation*}
\bar{r}\left(t_{1} t_{2}\right)=\frac{1}{|\mathbb{E}|} \sum_{\mathcal{G} \in \mathbb{E}} r\left(t_{1} t_{2}, \mathcal{G}\right) \tag{3.27}
\end{equation*}
$$

where $|\mathbb{E}|$ is the number of genomes in $\mathbb{E}$ and $\tilde{r}\left(t_{1} t_{2}\right)$ is the median of the observed/theoretical ratios $r\left(t_{1} t_{2}, \mathcal{G}\right)$ of a trinucleotide pair $t_{1} t_{2} \in B^{6}$ in the gene sequences of eukaryotes $\mathbb{E}$.

Remark 3.6. The three observed/theoretical ratios $r\left(t_{1} t_{2}\right), \bar{r}\left(t_{1} t_{2}\right)$, and $\tilde{r}\left(t_{1} t_{2}\right)$ of a trinucleotide pair $t_{1} t_{2} \in B^{6}$ have the same following statistical property. When they are greater than 1 , the trinucleotide pair $t_{1} t_{2}$ is over-represented in the eukaryotic gene sequences, and conversely when they below 1 .

The three observed/theoretical ratios $r\left(t_{1} t_{2}\right), \bar{r}\left(t_{1} t_{2}\right)$, and $\tilde{r}\left(t_{1} t_{2}\right)$ of trinucleotide pairs will lead to the same statistical results with circular code $X$ (see section 4.5).

### 3.7 Summary

We presented in this chapter the two types of data we have, which separated this work into several studies along the type of the data and how the data was handled. During this thesis we were able to develop a new algorithm that can search for motifs in any nucleic acid sequences, this algorithm is generic by being able to use any code, be it $X$ circular code, bijective transformation codes or randomly generated codes.

The first study focused on the relatively smaller sequences of rRNA and tRNA found in ribosomes, it covered seven organisms belonging to bacteria, archaea and eukaryote. We used the biinfinite word in search of motifs here using the Frames algorithm with a minimum length of 9 nucleotides and no restriction on the number of unique trinucleotides (cardinal) from $X$. The aim of the study was to study spatial significance of the $X$ motifs.

The second study dealt with huge amount of data, the complete chromosomes of 138 genomes, obtained from RefSeq (Table 3.2), which was coupled with the mapping of their coding region, obtained from GenBank. This study aimed to find statistical significance of the $X$ motifs compared with random sets, bijective transformations of $X$ and its first and second permuted codes ( $X_{1}$ and $X_{2}$ ). The motifs here collected with a minimum length of 10 trinucleotides (30 nucleotides) and a minimum of 5 unique trinucleotides.

The third study also was conducted on the complete chromosomes of 138 genomes, but it focused on searching for simple repeats which are in fact unitary circular codes. This required a new algorithm that retrieves the occurrence of each repeat in eukaryotic genomes.

The last and fourth study examines the pairing of trinucleotides in gene sequences of the eukaryotic genomes. This was conducted by using the GenBank annotation files to retrieve all gene sequences from a chromosome and study it separately in contrast to the second study where coding regions were drawn on the chromosome.

In the following chapter we will detail the results obtained from these studies while discussing their significance.

## 4

## Results and Discussion

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### 4.1 Introduction

This chapter will be divided into two sections along the data type and biological environment mentioned before.

The first section will show the results obtained from the spatial study of the circular code motifs found in the ribosome, whether in the important decoding center or around that region, along with a study for the tRNA sequences.

The second section presents a comparative study of the $X_{0}$ circular code against its 2 nd and 3rd permutations, $X_{1}$ and $X_{2}$ respectively, the 23 bijective circular code transformations and 30 random generated codes. We will show the importance and uniqueness of the $X_{0}$ circular code.

The third section involves the study of simple repeats and their ties to unitary circular codes, and a comparison between various sizes of a repeated word, dinucleotide, trinucleotide and tetranucleotide.

The fourth section presents the result from a study on the trinucleotide pairs in gene sequences of the eukaryotic genomes and the significance of identical trinucleotide pairs.

## 4.2 $X$ Circular code motifs in the ribosome

The data used in this study are presented in section 3.2.1 and the algorithm used to process this data is shown in section 3.2. The tools mentioned in section 3.4 were used to further analyse the sequences and the 3D structure of the ribosome.

### 4.2.1 $X$ CIRCULAR CODE MOTIFS IN THE RIBOSOMAL DECODING CENTER

The universally conserved nucleotides A1492 and A1493 have an experimentally proven biological function in the codon-anti-codon binding of tRNA as the A-site in the ribosome (Moazed and Noller, 1990; Powers and Noller, 1994; Yoshizawa, Fourmy, and Puglisi, 1999). Unexpectedly, the A1492 and A1493 nucleotides that belong to the ribosomal decoding center were found in $X$ circular code motifs.

### 4.2.1.1 The conserved A1492 and A1493 nucleotides

We will detail the $X$ motifs containing the universally conserved A1492 and A1493 nucleotides.

Table 4.1: Identification of $X$ circular code motifs $m_{A A}$ containing the universally conserved nucleotides A1492 and A1493 (in bold) in all studied rRNAs of bacteria, archaea, nuclear eukaryotes, and chloroplasts.

| PDB ID | Kingdom | Organism | X motif $\left(m_{A A}\right)$ | Start | End | Length |
| :--- | :--- | :--- | :--- | ---: | ---: | ---: |
| 3J5T | Bacteria | E. coli | G,GGT,GAA,GTC,GTA,AC | 1487 | 1501 | 15 |
| 3I8G | Bacteria | T. thermophilus | G,GAA,GGT,GC | 1490 | 1498 | 10 |
| 3J20 | Archaea | P. furiosus | A,GAA,GTC,GTA,AC | 1445 | 1456 | 9 |
| 3IZE | Eukaryote <br> (nuclear) | S. cerevisiaie | AA,GTC,GTA,AC | 1755 | 1764 | 10 |
| 3J5Z | Eukaryote <br> (nuclear) | T. aestivum | A,GAA,GTC,GTA,AC | 1763 | 1774 | 12 |
| 3J3D | Eukaryote <br> (nuclear) | H. sapiens | AA,GTC,GTA,AC | 1824 | 1833 | 10 |
| 3BBN | Eukaryote <br> (chloroplast) | S. oleracea | GT,GAA,GTC,GTA,AC | 1438 | 1450 | 13 |

### 4.2.1.1.1 The A1492 and A1493 nucleotides in $X$ motifs in bacterial rRNA

In the rRNAs of both $E$. coli and T. thermophilus, the conserved nucleotides A1492 and A1493 occur at the 2nd and 3rd sites of the trinucleotide $G A A \in$ $X$. In rRNA of $E$. coli, it belongs to the X motif $m_{A A}$ (E. coli, 1487, 1501, $15)=G, G G T, G A A, G T C, G T A, A C$ of 15 nucleotide length starting with the nucleotide G suffix of $C A G, C T G, G A G \in X$ followed by four trinucleotides $G A A, G G T, G T A, G T C \in X$ (given in lexicographical order) and ending with the dinucleotide AC prefix of $A C C \in X$. In rRNA of $T$. thermophilus, it belongs to the $X$ motif $m_{A A}(T$. thermophilus, 1490, 1498, 9) $=G, G \boldsymbol{A} \boldsymbol{A}, G G T, G C$ of nine nucleotide length starting with the nucleotide $G$, as in $E$. coli, followed by two trinucleotides $G A A, G G T \in X$ and ending with the dinucleotide $G C$ prefix of $G C C \in X$.

These two rRNA X motifs $m_{A A}\left(E\right.$. coli) and $m_{A A}(T$. thermophilus) have completely different primary structures. Thus, the classical bioinformatics methods, such as sequence alignment or phylogenetic inference, are not able to identify these motifs which were only revealed by the circular code theory.

In the rRNA of $T$. thermophilus, the following $X$ motif which we will call $m_{A A}^{*}(T$. thermophilus, $1461,1475,15)=G, G G C, G A A, G T C, G T A, A C$ of $15 \mathrm{nu}-$ cleotide length is aligned with the $X$ motif $m_{A A}$ ( $E$. coli) with only one nucleotide difference ( $T$ in $G G T$ replaced by $C$ in $T$. thermophilus). However, the $X$ motif $m_{A A}^{*}(T$. thermophilus) has a spatial structure far from the decoding center and probably has no function in modern rRNA of $T$. thermophilus.


Figure 4.1: X circular code motifs involved in the bacterial ribosome decoding center of Escherichia coli (crystallographic structure PDB 3J5T): the mRNA $X$ motifs (green), the rRNA $X$ motif $m_{A A}$ (E. coli, 1487, 1501, 15) (purple with the conserved A1492 and A1493 nucleotides in red), the rRNA $X$ motif $m_{G}$ (E. coli, 527, 536, 10) (orange with the conserved nucleotide G530 in fuchsia), the rRNA $X$ motif m(E. coli, 1396, 1404, 9) (yellow) and the tRNA $X$ motifs (blue with the anti-codon in black). The remaining rRNA (lemonchiffon) is outside the neighborhood of these $X$ motifs.

### 4.2.1.1.2 The A1492 and A1493 nucleotides in $X$ motifs in archaea rRNA

In the rRNA of P. furiosus, the conserved A1429 and A1493 nucleotides occur at the 2 nd and 3 rd sites of the trinucleotide $G A A \in X$ and belongs to the $X$ motif $m_{A A}(P$. furiosus, $1445,1456,12)=A, G \boldsymbol{A} \boldsymbol{A}, G T C, G T A, A C$ of 12 nucleotide length starting with the nucleotide $A$ suffix of $G A A, G T A \in X$ and then with a suffix of 11 nucleotides identical to the $X$ motif $m_{A A}$ (E. coli).

### 4.2.1.1.3 The A1492 and A1493 nucleotides in $X$ motifs in nuclear eukaryotic rRNA

There are significant differences between prokaryotic and eukaryotic rRNAs, in particular eukaryotic 18 S rRNAs are about $40 \%$ larger than the prokaryotic 16 S rRNAs. Nevertheless, some rRNA sites are conserved. In particular, the universally conserved A1429 and A1493 nucleotides of bacterial rRNAs occur in


Figure 4.2: X circular code motifs involved in the bacterial ribosome decoding center of Thermus thermophilus (crystallographic structure PDB 3I8G): the mRNA $X$ motifs (green), the rRNA $X$ motif $m_{A A}$ (T. thermophilus, $1490,1498,9)$ (purple with the conserved A1492 and A1493 nucleotides in red), the rRNA $X$ motif $m_{G}(T$. thermophilus, $528,536,9$ ) (orange with the conserved nucleotide $G 530$ in fuchsia), the rRNA $X$ motif m(T. thermophilus, $1375,1383,9$ ) (yellow) and the tRNA $X$ motifs (blue with the anti-codon in black). The remaining rRNA (lemonchiffon) is outside the neighbourhood of these $X$ motifs.
eukaryotic rRNAs but at different positions: 1755 and 1756 in S. cerevisiae, 1765 and 1766 in T. aestivum (Fan-Minogue and Bedwell, 2007) and 1824 and 1825 in H. sapiens (Bulygin et al., 2009).

In the rRNA of S. cerevisiae and H. sapiens, the conserved A1492 and A1493 nucleotides are the prefix of the X motif $m_{A A}(S$. cerevisiae, $1755,1764,10)=$ $m_{A A}(H$. sapiens, $1824,1833,10)=\boldsymbol{A} \boldsymbol{A}, G T C, G T A, A C$ of 10 nucleotides length followed by two trinucleotides $G T A, G T C \in X$ and ending with the dinucleotide $A C$ prefix of $A C C \in X$. In rRNA of $T$. aestivum, they occur occur at the 2 nd and 3rd sites of the trinucleotide $G A A \in X$ and belong to the $X$ motif $m_{A A}(T$. aestivum, $1763,1774,12)=A, G \boldsymbol{A} \boldsymbol{A}, G T C, G T A, A C$ of 12 nucleotides length which is identical to the archaeal $X$ motif $m_{A A}(P$. furiosus $)$.


Figure 4.3: X circular code motifs involved in the archaeal ribosome decoding center of Pyrococcus furiosus (crystallographic structure PDB 3J20): the rRNA $X$ motif $m_{A A}(P$. furiosus, $1445,1456,12$ ) (purple with the conserved A1492 and A1493 nucleotides in red), the rRNA $X$ motif $m_{G}$ (P.furiosus, 480, 497, 18) (orange with the conserved nucleotide G530 in fuchsia), the rRNA $X$ motif $m$ (P. furiosus, 1356, 1364, 9) (yellow) and the tRNA $X$ motifs (blue with the anti-codon in black). The remaining rRNA (lemonchiffon) is outside the neighbourhood of these $X$ motifs and the mRNA is missing (Table 3.1).

### 4.2.1.1.4 The A1492 and A1493 nucleotides in $X$ motifs in chloroplast rRNA

In the rRNA of $S$. oleracea, the conserved A1429 and A1493 nucleotides occur at the 2 nd and 3 rd sites of the trinucleotide $G A A \in X$ and belong to the $X$ motif $m_{A A}(S$. oleracea, $1438,1450,13)=G T, G \boldsymbol{A} \boldsymbol{A}, G T C, G T A, A C$ which is a suffix of 13 nucleotides of the bacterial $X$ motif $m_{A A}(E$. coli).

### 4.2.1.1.5 Summary of the A1492 and A1493 nucleotides in $X$ motifs

In all the studied rRNAs, the universally conserved nucleotides A1492 and A1493 precede the trinucleotide $G T C \in X$, except in $T$. thermophilus where $G T C$ is replaced by $G G T$. Thus, it always occurs at the 2 nd and 3rd sites of a trinucleotide which is always $G A A$ when the trinucleotide belongs to $X$. For one $X$ motif $m_{A A}(S$. cerevisiae $)=m_{A A}(H$. sapiens $)$, it is a suffix of $X$.

Figures 4.1-4.7 show that the rRNA $X$ motifs $m_{A A}(E$. coli $), m_{A A}(T$. ther-


Figure 4.4: X circular code motifs involved in the nuclear eukaryotic ribosome decoding center of Saccharomyces cerevisiae (crystallographic structure PDB 3IZE): the mRNA $X$ motifs (green), the rRNA $X$ motif $m_{A A}($ S. cerevisiae, $1755,1764,10)$ (purple with the conserved A1492 and A1493 nucleotides in red), the rRNA $X$ motif $m_{G}(S$. cerevisiae, $574,582,9)$ (orange with the conserved nucleotide $G 530$ in fuchsia), the rRNA $X$ motif m (S. cerevisiae, $1633,1641,9$ ) (yellow) and the tRNA $X$ motifs (blue with the anti-codon in black). The remaining rRNA (lemonchiffon) is outside the neighbourhood of these $X$ motifs.
mophilus), $m_{A A}(P$. furiosus $), m_{A A}(S$. cerevisiae $), m_{A A}(T$. aestivum $), m_{A A}(H$. sapiens), and $m_{A A}(S$. oleracea) (purple with the conserved dinucleotide AA in red) of bacteria, archaea, nuclear eukaryotes and chloroplasts belong to the ribosome decoding center with spatial relations with mRNA (green) and tRNA $X$ motifs (blue with the anti-codon in black). Which allows us to observe possible interactions between the $m_{A A} X$ circular code motifs from the rRNA and the tRNA and mRNA sequences.

### 4.2.1.2 The conserved G530 nucleotide

We will detail the motifs containing the conserved G530 nucleotide. Unlike in the prokayotic organisms studied, the G530 was not identified experimentally in eukaryotes. We will show the potential G530 nucleotide that is responsible for that role in eukaryotes (which was not identified yet


Figure 4.5: X circular code motifs involved in the nuclear eukaryotic ribosome decoding center of Triticum aestivum (crystallographic structure PDB 3J5Z): the mRNA (green), the rRNA $X$ motif $m_{A A}$ (T. aestivum, 1763, 1774,12 ) (purple with the conserved A1492 and A1493 nucleotides in red), the rRNA $X$ motif $m_{G}$ (T.aestivum, $578,586,9)$ (orange with the conserved nucleotide $G 530$ in fuchsia), the rRNA $X$ motif $m(T$. aestivum, 1641, 1649, 9) (yellow) and the tRNA $X$ motifs (blue with the anti-codon in black). The remaining rRNA (lemonchiffon) is outside the neighbourhood of these $X$ motifs.

## experimentally) using circular code theory.

### 4.2.1.2.1 The G530 nucleotide in $X$ motifs in bacterial rRNA

In the rRNA of $E$. coli, the conserved nucleotide G occurs at the 2nd site of the trinucleotide $G G T \in X$ and belongs to the $X$ motif $m_{G}(E$. coli, $527,536,10)=$ $G C, G G T, A A T, A C$ of 10 nucleotide length starting with the dinucleotide $G C$ suffix of $G G C \in X$ followed by two trinucleotides $A A T, G G T \in X$ and ending with the dinucleotide $A C$ prefix of $A C C \in X$. In the rRNA of $T$. thermophilus, the conserved nucleotide $G$ occurs at the 1st site of $G T T \in X$ and belongs to the $X$ motif $m_{G}(T$. thermophilus, $528,536,9)=G C, G T T, A C C, C$ of nine nucleotide length starting with the dinucleotide $G C$, as in $E$. coli, followed by two trinucleotides $A C C, G T T \in X$ and ending with the nucleotide $C$ prefix of $C A G, C T C, C T G \in X$. As with the conserved A1492 and A1493 nucleotides,

Table 4.2: Identification of $X$ circular code motifs $m_{G}$ containing the conserved nucleotide $G 530$ (in bold) in rRNAs of bacteria and archaea. The bottom half of the table shows the $X$ circular code motifs potentially containing the equivalent G530 in nuclear eukaryotes and chloroplasts.

| PDB ID | Kingdom | Organism | $\mathbf{X}$ motif $\left(m_{G}\right)$ | Start | End | Length |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 J 5 T | Bacteria | E. coli | GC,GGT,AAT,AC | 527 | 536 | 10 |
| 3I8G | Bacteria | T. thermophilus | GC,GTT,ACC,C | 528 | 536 | 9 |
| 3 J 20 | Archaea | P. furiosus | GC,GGT,AAT,ACC,GGC,GGC,C | 480 | 497 | 18 |
| 3IZE | Eukaryote (nuclear) | S. cerevisiaie | GC,GGT,AAT, T | 574 | 582 | 9 |
| 3 J 5 Z | Eukaryote (nuclear) | T. aestivum | GC,GGT,AAT, T | 578 | 586 | 9 |
| 3J3D | Eukaryote (nuclear) | H. sapiens | GC,GGT,AAT, T | 623 | 631 | 9 |
| 3 BBN | Eukaryote (chloroplast) | S. oleracea | GC,GGT,AA | 475 | 481 | 7 |

the two rRNA $X$ motifs $m_{G}\left(E\right.$. coli) and $m_{G}(T$. thermophilus) have completely different primary structures and can only be revealed by the circular code theory.

### 4.2.1.2.2 The G530 nucleotide in $X$ motifs in archaea rRNA

In rRNA of $P$.furiosus, the conserved nucleotide $G$ occurs at the 2 nd site of the trinucleotide $G G T \in X$, as in $m_{G}(E$. coli), and belongs to the $X$ motif $m_{G}(P$. furiosus, $480,497,18)=G C, G G T, A A T A C C, G G C, G G C, C$ of $18 \mathrm{nu}-$ cleotide length starting with the dinucleotide $G C$, as in $m_{G}\left(E\right.$. coli) and $m_{G}(T$. thermophilus), followed by five trinucleotides $A A T, A C C, G G C, G G T \in X$ and ending with the nucleotide $C$, as in $m_{G}(T$. thermophilus).

### 4.2.1.2.3 The G530 nucleotide in $X$ motifs in eukaryotic rRNA

By applying our modifications mentioned in section 3.4.1 on the results of the global multiple sequence alignment ClustalX, the $X$ motifs $m_{G}$ are found in rRNAs of nuclear eukaryotes $S$. cerevisiae, T. aestivum, H. sapiens and chloroplasts $S$. oleracea. Very surprisingly, a common $X$ motif $m_{G}$ is identified in rRNAs of nuclear eukaryotes: $m_{G}(\mathrm{~S}$. cerevisiae, $574,582,9)=m_{G}(\mathrm{~T}$. aestivum, 578, 586, 9) $=m_{G}$ (H. sapiens, 623, 631, 9) $=m_{G}$ (Nuclear eukaryotes) $=G C, G G T, A A T, T$ (Table 4.2). Furthermore, a $X$ motif $m_{G}$ is also identified in rRNA of chloroplasts: $m_{G}($ S. oleracea, $475,481,7)=m_{G}$ (Chloroplasts) $=$ $G C, G G T, A A$ (Table 4.2) which is a prefix of seven nucleotides of $m_{G}$ (Nuclear eukaryotes). As the common $X$ motif $m_{G}$ (Nuclear eukaryotes, Chloroplasts) $=$ $G C, G G T, A A$ is a prefix of the common $X$ motif $m_{G}(E$. coli, P. furiosus $)=$ $G C, G G T, A A T, A C$, we can make the realistic hypothesis that the conserved


Figure 4.6: Xcircular code motifs involved in the nuclear eukaryotic ribosome decoding center of Homo sapiens (crystallographic structure PDB 3J3D): the rRNA $X$ motif $m_{A A}$ (H. sapiens, 1824, 1833, 10) (purple with the conserved A1492 and A1493 nucleotides in red), the rRNA $X$ motif $m_{G}$ (H. sapiens, 623, 631, 9) (orange with the conserved nucleotide G530 in fuchsia), the rRNA $X$ motif $m$ (H. sapiens, 1697, 1705, 9) (yellow) and the tRNA $X$ motifs (blue with the anti-codon in black). The remaining rRNA (lemonchiffon) is outside the neighbourhood of these $X$ motifs and the mRNA is missing (Table 3.1).
nucleotide $G$ in nuclear and chloroplast rRNAs occurs at the 2nd site of the trinucleotide $G G T \in X$.

Furthermore, Figures 4.4-4.7 show that the rRNA $X$ motifs $m_{G}$ (S. cerevisiae), $m_{G}\left(T\right.$. aestivum), $m_{G}(H$. sapiens $)$ and $m_{G}(S$. oleracea) (orange with the conserved nucleotide G in fuchsia) of nuclear eukaryotes and chloroplasts belong to the ribosome decoding center with spatial relations with mRNA (green) and tRNA $X$ motifs (blue with the anti-codon in black).

### 4.2.1.2.4 Summary of the G530 nucleotide in $X$ motifs

The bacterial $X$ motif $m_{G}(E$. coli $)$ is a prefix of 10 nucleotides of the archaeal $X$ motif $m_{G}(P$. furiosus). The conserved nucleotide G530 occurs at the 2nd site of $G G T \in X$ in $m_{G}(E$. coli $)$ and $m_{G}(P$. furiosus $)$, and at the 1 st site of $G T T \in X$ in $m_{G}(T$. thermophilus). Figures 4.1-4.3 show that the rRNA $X$


Figure 4.7: X circular code motifs involved in the chloroplast eukaryotic ribosome decoding center of Spinacia oleracea (crystallographic structure PDB 3BBN): the rRNA $X$ motif $m_{A A}$ (S. oleracea, 1438, 1450, 13) (purple with the conserved A1492 and A1493 nucleotides in red), the rRNA $X$ motif $m_{G}(S$. oleracea, 475, 481, 7) (orange with the conserved nucleotide G530 in fuchsia) and the rRNA $X$ motif $m(S$. oleracea, 1345, 1353, 9) (yellow). The remaining rRNA (lemonchiffon) is outside the neighbourhood of these $X$ motifs, the mRNA and tRNA are missing (Table 3.1).
motifs $m_{G}\left(E\right.$. coli), $m_{G}(T$. thermophilus $)$ and $m_{G}(P$. furiosus) (orange with the conserved nucleotide $G$ in fuchsia) of bacteria and archaea belong to the ribosome decoding center with spatial relations with mRNA (green) and tRNA $X$ motifs (blue with the anti-codon in black).

Using prior knowledge of the G530 in bacteria, we were able to identify a possible location of its equivalent in eukaryotes. This was based on similarities of motifs in terms of composition and their spatial location in the ribosome when comparing between prokaryotes and eukaryotes. The eukaryotic $X$ motifs $m_{G}$ are identical for the nuclear eukaryotes $G C, G G T, A A T, T$, where as the chloroplast $X$ motif $m_{G}$ is two nucleotides shorter having possibly lost the third nucleotide $T$ of the trinucleotide $A A T \in X$ to mutation same as the following $T$ nucleotide (Table 4.2).

Table 4.3: Identification of a conserved $X$ circular code motifs $m$ in all studied rRNAs of bacteria, archaea, nuclear eukaryotes, and chloroplasts

| PDB ID | Kingdom | Organism | X motif ( $m$ ) | Start | End | Length |
| :--- | :--- | :--- | :--- | ---: | ---: | ---: |
| 3J5T | Bacteria | E. coli | AC,ACC,GCC,C | 1396 | 1404 | 9 |
| 3I8G | Bacteria | T. thermophilus | AC,ACC,GCC,C | 1375 | 1383 | 9 |
| 3J20 | Archaea | P. furiosus | AC,ACC,GCC,C | 1356 | 1364 | 9 |
| 3IZE | Eukaryote <br> (nuclear) | S. cerevisiaie | AC,ACC,GCC,C | 1633 | 1641 | 9 |
| 3J5Z | Eukaryote <br> (nuclear) | T. aestivum | AC,ACC,GCC,C | 1641 | 1649 | 9 |
| 3J3D | Eukaryote <br> (nuclear) | H. sapiens | AC,ACC,GCC,C | 1697 | 1705 | 9 |
| 3BBN | Eukaryote <br> (chloroplast) | S. oleracea | AC,ACC,GCC,C | 1345 | 1353 | 9 |

### 4.2.2 Spatial study of $X$ circular code motifs near the ribosomal DECODING CENTER

At this stage we shifted our region of interest to around the decoding center to examine possible conserved $X$ motif in the vicinity of mRNA and tRNAs that could have a possible role in the translation process by interaction.

### 4.2.2.1 A CONSERVED $X$ motif near the ribosomal decoding center

We were able to identify an $X$ motif $m$ which is universally conserved in rRNAs of bacteria, archaea, nuclear eukaryotes, and chloroplasts (Table 4.3): $m=$ AC,ACC,GCC,C of nine nucleotide length starting with the dinucleotide AC suffix of AAC,GAC, TAC $\in X$ followed by two trinucleotides ACC,GCC $\in X$ and ending with the nucleotide C prefix of CAG,CTC,CTG $\in X$. The start and end positions of the $X$ motif $m$ in the seven studied organisms are given in Table 4.3. Very unexpectedly, Figures 4.1-4.7 show that the universally conserved rRNA $X$ motif $m$ (yellow) of bacteria, archaea, nuclear eukaryotes, and chloroplasts belongs to the ribosome decoding center with spatial relations with mRNA (green) and tRNA X motifs (blue with the anti-codon in black). With the sole exception being the T. thermophilus where this conserved motif is located outside the decoding center, but as mentioned before this organism has several differences in terms of structure than the others.

Table 4.4: Identification of seven $X$ circular code motifs $\operatorname{Pr} R N A X m$ in 16 SrRNAs of prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus) near the ribosome decoding center.

| Alias | $X$ circular code motif | Organism | Start | End | Length |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{PrRNAX} m_{1}$ | G, GAG, GGT, GC | E. coli (3J5T) | 537 | 545 | 9 |
|  | G, GAG, GGC, GC | T. thermophilus (3I8G) | 517 | 525 | 9 |
|  | GC, GGT, AAT, ACC, GGC, GGC , C | P. furiosus (3J20) | 480 | 497 | 18 |
| $\operatorname{PrRNAXm} \mathrm{m}_{2}$ | GC, GGT, GAA, AT | E. coli (3J5T) | 688 | 697 | 10 |
|  | GC, GGT, GAA, AT | T. thermophilus (318G) | 668 | 677 | 10 |
|  | G, GGT, GAA, ATC, CT | P. furiosus (3J20) | 643 | 654 | 12 |
| $\operatorname{PrRNAXm} 3$ | G, AAT, ACC, GGT, GGC, GAA, GGC, GGC, C | E. coli (3J5T) | 714 | 736 | 23 |
|  | G, AAC, GCC, GAT, GGC, GAA, GGC, A | T. thermophilus (318G) | 694 | 713 | 20 |
|  | GT, GGC, GAA , GGC, GCC, C | P. furiosus (3J20) | 676 | 690 | 15 |
| $\operatorname{PrRNAXm} 4$ | TA, GAT, ACC, CTG, GTA, GTC, CA | E. coli (3J5T) | 789 | 807 | 19 |
|  | TA, GAT, ACC, C | T. thermophilus (318G) | 769 | 777 | 9 |
|  | TA, GAT, ACC, C | P. furiosus (3J20) | 743 | 751 | 9 |
| $\operatorname{PrRNAXm}{ }_{5}$ | G, GAT, GAC, GTC, AA | E. coli (3J5T) | 1186 | 1197 | 12 |
|  | G, GAC, GAC, GTC, T | T. thermophilus (3I8G) | 1164 | 1174 | 11 |
|  | G, GGC, GAC, GGT, A | P. furiosus (3J20) | 1146 | 1188 | 9 |
| $\operatorname{PrRNAXm}{ }_{6}$ | T,TAC, GAC, CAG, GGC, TAC, AC | E. coli (3J5T) | 1211 | 1228 | 18 |
|  | T, TAC, GGC, CTG, GGC, GAC, AC | T. thermophilus (3I8G) | 1189 | 1206 | 18 |
|  | G, GGC, TAC, AC | P. furiosus (3J20) | 1180 | 1188 | 9 |
| $\operatorname{PrRNAXm} m_{7}$ | AC, GGT, GAA, TAC, GTT, C | E. coli (3J5T) | 1368 | 1382 | 15 |
|  | GC, GGT, GAA, TAC, GTT, C | T. thermophilus (3I8G) | 1347 | 1361 | 15 |
|  | GC, GGC, GAA , TAC, GTC, C | P. furiosus (3J20) | 1328 | 1342 | 15 |

### 4.2.2.2 Conserved $X$ motifs in the rRNA of prokaryotes

The circular code theory identified seven $X$ circular code motifs, $\operatorname{Pr} R N A X m$, that are conserved in the prokaryotic 16 s rRNA of bacteria E. coli(3J5T) and $T$. thermophilus(3I8G), and archaea P. furiosus(3J20) (Table 4.4).
(i) $\operatorname{PrRNAX} m_{1}(E$. coli, 537, 545, 9) $=\operatorname{PrRNAXm} 1$ (T. thermophilus, 517, $525,9)=G, G A G, G G Y, G C$ of nine nucleotides starts with the nucleotide $G$ suffix of $\{C A G, C T G, G A G\} \in X$, has two trinucleotides $G A G, G G Y \in$ $X$ where $Y=T$ in $E$. coli and $Y=C$ in $T$. thermophilus, and ends with the dinucleotide $G C$ prefix of $G C C \in X$. The large X motif $\operatorname{PrRNAXm} m_{1}(P$. furiosus, 480, 497, 18) $=G C, G G T, A A T, A C C, G G C, G G C, C$ of $18 \mathrm{nu}-$ cleotides starts with the dinucleotide $G C$ suffix of $G G C \in X$, has five trinucleotides $G G T, A A T, A C C, G G C, G G C \in X$ and ends with the nucleotide $C$ prefix of $\{C A G, C T C, C T G\} \in X . \operatorname{Pr} R N A X m_{1}$ of $E$. coli and T. thermophilus are partial suffixes of $\operatorname{Pr} R N A X m_{1}$ of $P$. furiosus.
(ii) $\operatorname{Pr} R N A X m_{2}$ (E. coli, 688, 697, 10), $\operatorname{Pr} R N A X m_{2}(T$. thermophilus, 668, 677, 10) and $\operatorname{Pr} R N A X m_{2}(P$. furiosus, 643, 654, 12) have the com-


Figure 4.8: $\quad X$ circular code motifs near the bacterial ribosome decoding center of Escherichia coli (PDB 3J5T): the mRNA (green), the rRNA $X$ motifs $\operatorname{Pr} R N A X m_{1}$ (537, 545, 9) (maroon), $\operatorname{Pr} R N A X m_{2}$ (688, 697, 10) (pink), $\quad \operatorname{Pr} R N A X m_{3}$ (714, 736, 23) (gold), $\quad \operatorname{Pr} R N A X m_{4}(789, \quad 807, \quad 19) \quad$ (orange), $\operatorname{Pr} R N A X m_{5}(1186, \quad 1197,12)$ (navy), $\operatorname{Pr} R N A X m_{6}(1211,1228,18) \quad$ (purple), $\operatorname{Pr} R N A X m_{7}(1368,1382,15)$ (red), and the tRNA ( 50 region in dark blue, 30 region in clearer blue and anti-codon in black). The remaining rRNA (lemonchiffon) is outside the neighborhood of these $X$ motifs.


Figure 4.9: $X$ circular code motifs near the bacterial ribosome decoding center of Thermus thermophilus (PDB ID: 3I8G): the mRNA (green), the rRNA $X$ motifs $\operatorname{Pr} R N A X m_{1}$ (517, 525, 9) (maroon), $\operatorname{Pr} R N A X m_{2}$ (668, 677, 10) (pink), $\quad \operatorname{Pr} R N A X m_{3}$ (694, 713, 20) (gold), $\operatorname{PrRNAXm_{4}(769,~777,~9)~(orange),~}$ $\operatorname{Pr} R N A X m_{5}$ (1164, 1174, 11) (navy), $\operatorname{Pr} R N A X m_{6}$ (1189, 1206, 18) (purple), $\operatorname{Pr} R N A X m_{7}(1347,1361,15)$ (red), and the tRNA ( 50 region in dark blue, 30 region in clearer blue and anti-codon in black). The remaining rRNA (lemonchiffon) is outside the neighborhood of these $X$ motifs.
mon $X$ motif $G G T, G A A, A T$ of eight nucleotides with $G G T, G A A \in X$. $\operatorname{Pr} R N A X m_{2}$ of $E$. coli and T. thermophilus start with the dinucleotide $G C$ suffix of $G G C \in X$ and end with the dinucleotide AT prefix of $\{A T C, A T T\} \in X . \operatorname{Pr} R N A X m_{2}$ of $P$. furiosus starts with the nucleotide G suffix of $\{C A G, C T G, G A G\} \in X$ and ends with the dinucleotide $C T$ prefix of $\{C T C, C T G\} \in X$.
(iii) $\operatorname{Pr} R N A X m_{3}\left(E . \quad\right.$ coli, 714, 736, 23) and $\operatorname{PrRNAXm} m_{3}(T$. thermophilus, 694, 713, 20) have the large common $X$ motif $G, A A Y, R_{1} C C, G R_{2} T, G G C, G A A, G G C$ of 19 nucleotides starting with the nucleotide $G$ suffix of $\{C A G, C T G, G A G\} \in X$ followed by six trinucleotides $A A Y, R_{1} C C, G R_{2} T, G G C, G A A, G G C \quad \in \quad X$ where $\mathrm{Y}=\mathrm{T}, \mathrm{R}_{1}=\mathrm{A}$ and $\mathrm{R}_{2}=\mathrm{G}$ in $E$. coli, while $\mathrm{Y}=\mathrm{C}, \mathrm{R}_{1}$ $=\mathrm{G}$ and $\mathrm{R}_{2}=\mathrm{A}$ in T. thermophilus. $\operatorname{Pr} R N A X m_{3}$ of E. coli ends with the nucleotide C prefix of $\{C A G, C T C, C T G\} \in X$ whereas
$\operatorname{PrRNAX} m_{3}$ of $T$. thermophilus ends with the nucleotide A prefix of $\{A A C, A A T, A C C, A T C, A T T\} \in X$. The X motif $\operatorname{Pr} R N A X m_{3}$ (P. furiosus, $676,690,15)=G T, G G C, G A A, G G C, G C C, C$ of 15 nucleotides is a conserved suffix of $\operatorname{Pr} R N A X m_{3}$ of E. coli (14 identical letters among 15) starting with the dinucleotide GT suffix of $G G T$ in $\operatorname{Pr} R N A X m_{3}$ of E. coli.
(iv) The large X motif $\operatorname{PrRNAXm} m_{4}(E . \quad$ coli, 789, 807, 19) $=$ $T A, G A T, A C C, C T G, G T A, G T C, C A$ of 19 nucleotides starts with the dinucleotide $T A$ suffix of $G T A \in X$, has five trinucleotides $G A T, A C C, C T G, G T A, G T C \in X$ and ends with the dinucleotide $C A$ prefix of $C A G \in X . \operatorname{Pr} R N A X m_{4}(T$. thermophilus, 769, 777, 9) $=\operatorname{PrRNAX} m_{4}(P$.furiosus, 743, 751, 9) $=T A, G A T, A C C, C$ of nine nucleotides is a prefix of $\operatorname{PrRNAX} m_{4}$ of $E$. coli ending with the nucleotide $C$ prefix of $C T G$ in $\operatorname{Pr} R N A X m_{4}$ of $E$. coli.
(v) $\operatorname{Pr} R N A X m_{5}\left(E\right.$. coli, 1186, 1197, 12), $\operatorname{Pr} R N A X m_{5}(T$. thermophilus, 1164, $1174,11)$ and $\operatorname{Pr} R N A X m_{5}(P$. furiosus, 1146, 1156, 11) have the common $X$ motif $G, G R Y_{1}, G A C, G K Y_{2}, W$ of 11 nucleotides starting with the nucleotide $G$ suffix of $\{C A G, C T G, G A G\} \in X$ followed by three trinucleotides $G R Y_{1}, G A C, G K Y_{2} \in X$ where $R=A, Y_{1}=T, K=T, Y_{2}=C$ and $W=A$ in E. coli, $R=A, Y_{1}=C, K=T, Y_{2}=C$ and $W=T$ in T. thermophilus while $R=G, Y_{1}=C, K=G, Y_{2}=T$ and $W=A$ in P. furiosus. $\operatorname{Pr} R N A X m_{5}$ of $E$. coli ends with the dinucleotide $A A$ prefix of $\{A A C, A A T\} \in X, \operatorname{PrRNAX} m_{5}$ of $T$. thermophilus ends with the nucleotide $T$ prefix of $\{T A C, T T C\} \in X$ and $\operatorname{Pr} R N A X m_{5}$ of $P$. furiosus ends with the nucleotide A prefix of $\{A A C, A A T, A C C, A T C, A T T\} \in X$.
(vi) The large common $X$ motif $\operatorname{Pr} R N_{A X} m_{6}(E . \quad$ coli, 1211, 1228, 18) $=\operatorname{Pr} R N A X m_{6}(T$. thermophilus, 1189, 1206, 18) $=$ $T, T A C, G R C, C W G, G G C, K A C, A C$ of 18 nucleotides starts with the nucleotide T suffix of $\{A A T, A T T, G A T, G G T, G T T\} \in X$, has five trinucleotides $T A C, G R C, C W G, G G C, K A C \in X$ where $R=A, W=A$ and $K=T$ in $E$. coli while $R=G, W=T$ and $K=G$ in $T$. thermophilus, and ends with the dinucleotide $A C$ prefix of $A C C \in X . \operatorname{Pr} R N A X m_{6}(P$. furiosus, 1180, 1188, 9) $=G, G G C, T A C, A C$ of nine nucleotides is a suffix of $\operatorname{PrRNAXm} m_{6}$ of $E$. coli starting with the nucleotide $G$ suffix of $C A G$ in


Figure 4.10: $X$ circular code motifs near the archaeal ribosome decoding center of Pyrococcus furiosus (PDB ID: 3J20): the rRNA Xmotifs $\operatorname{Pr} R N A X m_{1}(480$, 497, 18) (maroon), $\operatorname{PrRNAXm} m_{2}$ (643, 654, 12) (pink), $\operatorname{PrRNAX} m_{3}(676, ~ 690, ~ 15)$ (gold), $\operatorname{PrRNAX} m_{4}(743,751, ~ 9)$ (orange), $\operatorname{PrRNAX} m_{5}(1146, \quad 1156,11)$ (navy), $\operatorname{PrRNAX} m_{6}(1180, \quad 1188, \quad 9) \quad$ (purple), $\operatorname{PrRNAX} m_{7}(1328,1342,15)$ (red), and the tRNA ( 50 region in dark blue, 30 region in clearer blue and anti-codon in black). The remaining rRNA (lemonchiffon) is outside the neighbourhood of these $X$ motifs and the mRNA is missing (Table 3.1).


Figure 4.11: $X$ circular code motifs near the (nuclear) eukaryotic ribosome decoding center of Saccharomyces cerevisiae (PDB ID: 3IZE): the mRNA (green), the rRNA $X$ motifs $\operatorname{ErRNAXm_{1}(900,~}$ 911, 12) (purple), $\operatorname{ErRNAXm} m_{2}(987,1004$, 18) (pink), $\operatorname{ErRNAX} m_{3}(1189,1197,9)$ (red), $\operatorname{ErRNAX} m_{4}(1564,1582,19)$ (orange), and the tRNA (50 region in dark blue, 30 region in clearer blue and anti-codon in black). The remaining rRNA (lemonchiffon) is outside the neighbourhood of these $X$ motifs.
$\operatorname{PrRNAXm} m_{6}$ of E.coli.
(vii) The common $X$ motif $\operatorname{PrRNAXm} m_{7}(E$. coli, 1368, 1382, 15) $=$ $\operatorname{PrRNAX} m_{7}\left(T\right.$. thermophilus, 1347, 1361, 15) $=\operatorname{PrRNAX} m_{7}(P$. furiosus, $1328,1342,15)=R C, G G Y, G A A, T A C, G T Y, C$ of 15 nucleotides start with the dinucleotide $A C(R=A)$ suffix of $\{A A C, G A C, T A C\} \in$ $X$ in $E$. coli and with the dinucleotide $G C(R=G)$ suffix of $G G C \in X$ in $T$. thermophilus and $P$.furiosus, has four trinucleotides $G G Y, G A A, T A C, G T Y \in X$ where $Y=T$ in $E$. coli and $T$. thermophilus while $Y=C$ in $P$.furiosus, and ends with the nucleotide $C$ prefix of $\{C A G, C T C, C T G\} \in X$.

Figures 4.8-4.10 show the prokaryotic rRNA $X$ motifs $\operatorname{Pr} R N A X m_{1}$ in maroon, $\operatorname{Pr} R N A X m_{2}$ in pink, $\operatorname{Pr} R N A X m_{3}$ in gold, $\operatorname{Pr} R N A X m_{4}$ in orange, $\operatorname{Pr} R N A X m_{5}$ in navy blue, $\operatorname{PrRNAXm} m_{6}$ in purple and $\operatorname{Pr} R N A X m_{7}$ in red
of E. coli, T. thermophilus and $P$. furiosus are near the ribosome decoding center ( 50 regions of tRNAs in dark blue, 30 regions of tRNAs in clearer blue and anti-codons of tRNAs in black).


### 4.2.2.3 Conserved $X$ motifs in the rRNA of eukaryotes

The circular code theory identified four $X$ circular code motifs, $\operatorname{Pr} R N A X m$, that are conserved in the eukaryotic 18s rRNA of $S$. cerevisiae (3IZE), T. aestivum (3J5Z) and H. sapiens (3J3D) (Table 4.5)
(i) $\operatorname{ErRNAX} m_{1}\left(S\right.$. cerevisiae, 900, 911, 12) $=\operatorname{ErRNAX} m_{1}(T$. aestivum, 905, 916, 12) $=\operatorname{ErRNAXm} m_{1}(H$. sapiens, 957, 968, 12) $=$ $A, G G T, G A A, A T T, C T$ of 12 nucleotides starts with the nucleotide A suffix of $\{G A A, G T A\} \in X$, has three trinucleotides $G G T, G A A, A T T \in X$ and ends with the dinucleotide $C T$ prefix of $\{C T C, C T G\} \in X$.
(ii) The large common X motif $\operatorname{ErRNAXm} 2(S$. cerevisiae, 987, 1004, 18) $=\operatorname{ErRNAX} m_{2}\left(T\right.$. aestivum, 992, 1009, 18) $=\operatorname{ErRNAXm} m_{2}(H$. sapiens,

Table 4.5: Identification of four $X$ circular code motifs $E r R N A X m$ in 18 s rRNAs of (nuclear) eukaryotes (S. cerevisiae, T. aestivum, H. sapiens) near the ribosome decoding center.

| Alias | $X$ circular code motif | Organism | Start | End | Length |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{ErRNAX} m_{1}$ | A, GGT, GAA, ATT, CT | S. cerevisiae (3IZE) | 900 | 911 | 12 |
|  | A, GGT, GAA , ATT, CT | T. aestivum (3J5Z)) | 905 | 916 | 12 |
|  | A, GGT, GAA, ATT, CT | H. sapiens (3J3D) | 957 | 968 | 12 |
| $\operatorname{ErRNAX} m_{2}$ | G, ATC, GAA , GAT, GAT, CAG , AT | S. cerevisiae (3IZE) | 987 | 1004 | 18 |
|  | G, CTC, GAA , GAC, GAT, CAG, AT | T. aestivum (3J5Z)) | 662 | 1009 | 18 |
|  | G , TTC, GAA , GAC, GAT, CAG , AT | H. sapiens (3J3D) | 1044 | 1061 | 18 |
| $\operatorname{ErRNAX} m_{3}$ | A, CTC, AAC, AC | S. cerevisiae (3IZE) | 1189 | 1197 | 9 |
|  | A, CTC, AAC, AC | T. aestivum (3J5Z)) | 1193 | 1201 | 9 |
|  | A, CTC, AAC, AC | H. sapiens (3J3D) | 1246 | 1254 | 9 |
| $\operatorname{ErRNAX} m_{4}$ | TC, TTC, AAC, GAG, GAA , TTC , CT | S. cerevisiae (3IZE) | 1564 | 1582 | 19 |
|  | TC, AAC, GAG, GAA, T | T. aestivum (3J5Z)) | 1575 | 1596 | 12 |
|  | TG, AAC, GAG, GAA, TTC, C | H. sapiens (3J3D) | 1631 | 1645 | 15 |

1044, 1061, 18) $=G, N T C, G A A, G A Y, G A T, C A G, A T$ of 18 nucleotides starts with the nucleotide $G$ suffix of $\{C A G, C T G, G A G\} \in X$, has five trinucleotides $N T C, G A A, G A Y, G A T, C A G \in X$ where $N=A$ and $Y=T$ in $S$. cerevisiae, $N=C$ and $Y=C$ in $T$. aestivum, while $N=T$ and $Y=C$ in $H$. sapiens, and ends with the dinucleotide AT prefix of $\{A T C, A T T\} \in X$.
(iii) $\operatorname{ErRNAX} m_{3}(S$. cerevisiae, 1189, 1197, 9) $=\operatorname{ErRNAXm} 3$ (T. aestivum, 1193, 1201, 9) $=\operatorname{ErRNAXm} 3$ (H. sapiens, 1246, 1254, 9) $=$ $A, C T C, A A C, A C$ of nine nucleotides starts with the nucleotide $A$ suffix of $\{G A A, G T A\} \in X$, has two trinucleotides $C T C, A A C \in X$ and ends with the dinucleotide $A C$ prefix of $A C C \in X$.
(iv) The large X motif $\operatorname{ErRNAXm} m_{4}(S$. cerevisiae, 1564, 1582, 19) $=$ $T C, T T C, A A C, G A G, G A A, T T C, C T$ of 19 nucleotides starts with the dinucleotide $T C$ suffix of $\{A T C, C T C, G T C, T T C\} \in X$, has five trinucleotides $T T C, A A C, G A G, G A A, T T C \in X$ and ends with the dinucleotide $C T$ prefix of $\{C T C, C T G\} \in X . \operatorname{ErRNAX} m_{4}(T$. aestivum, $1575,1596,12)=T C, A A C, G A G, G A A, T$ is a factor of $E r R N A X m_{4}$ of $S$. cerevisiae starting with the dinucleotide $T C$ suffix of the 1st $T T C \in X$ in $E r R N A X m_{4}$ of $S$. cerevisiae and ending with the nucleotide T prefix of the 2nd TTC $\in X$ in $E r R N A X m_{4}$ of $S$. cerevisiae. The X motif $\operatorname{ErRNAXm} m_{4}(H$. sapiens, 1631, 1645, 15) $=$
$T G, A A C, G A G, G A A, T T C, C$ is an almost exact suffix of $E r R N A X m_{4}$ of $S$. cerevisiae.

Figures 4.11-4.13 show the (nuclear) eukaryotic rRNA X motifs Er $R N A X m_{1}$ in purple, $\operatorname{Er} R N A X m_{2}$ in pink, $E r R N A X m_{3}$ in red and $E r R N A X m_{4}$ in orange of $S$. cerevisiae, T. aestivum and $H$. sapiens are near the ribosome decoding center ( 50 regions of tRNAs in dark blue, 30 regions of tRNAs in clearer blue and anti-codons of tRNAs in black), except ErRNAXm $m_{1}$ in S. cerevisiae and T. aestivum.

### 4.2.3 $X$ CIRCULAR CODE MOTIFS IN PROKARYOTIC TRNAS

We give the main features of $X$ motifs for each isoaccepting tRNA of prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus), more details on the $X$ motifs are found in Tables 4.6-4.26. We use the classical genetic alphabet convention to be able to engulf $X$ motifs that are fairly similar within a unified pattern. Let $R=\{A, G\}, Y=\{C, T\}, S=\{C, G\}, W=\{A, T\}, K=\{G, T\}$, $M=\{A, C\}$ and $N=\{A, C, G, T\}$. Furthermore, in term of $X$ motif length, we are distinguishing three classes of $X$ motifs: very large $X$ motifs greater or equal to 20 nucleotides (remember that the average lengths of prokaryotic tRNAs range typically from 71 to 91 nucleotides for Cys and Ser, respectively, see Section 2.4.1 and Fig. 2 in Michel, 2013), large $X$ motifs between 16 and 19 nucleotides and $X$ motifs between 9 and 15 nucleotides. $X$ motifs of lengths equal to 9 nucleotides already retrieve the reading frame with a probability of $99.9 \%$ and $X$ motifs of lengths greater or equal to 12 nucleotides always retrieve, by definition, the reading frame, i.e. with a probability of $100 \%$ (Table 3 and Fig. 4 in Michel, 2012). Moreover, the underline in an $X$ motif signifies that the underlined nucleotides are in common with one or more other motifs.

Finally, the $X$ motifs are studied according to three regions of tRNAs: $X$ motifs between the 5 ' end of tRNAs and the anti-codon (called here 5 ' region), $X$ motifs having at least one nucleotide in the anti-codon (anti-codon regions) and $X$ motifs between the anti-codon and the 3 ' end of tRNAs ( 3 ' region). The results below will identify $X$ motifs, a few of them being very large, and different relations, in particular a shifting by $0,+1$ or $+2 \bmod 3$ nucleotides with other $X$ motifs or the anti-codon.

We would like to notify the reader that the following sections offer a exhaustive description of the $X$ circular code motifs in tRNA sequences are presented.

These are particularly important as a reference for future studies targeting tRNAs.

### 4.2.3.1 $X$ circular code motifs in Ala-tRNAs (Table 4.6)

(i) 5' region of Ala-tRNAs: The $X$ motif $T, C A G, C T G, G G$ and the class of $X$ motifs $G C, C T G, G W A, K$ are shifted in frame (modulo 3 according to their suffix-prefix). The class of $X$ motifs $G C, G C C, G C C, Y T$ occurs before (5') the anti-codons $G G C$ and $T G C$.
(ii) anti-codon regions of Ala-tRNAs: The $X$ motif $G C, G C C, G C C, C T C, G C$ is in a different frame than the anti-codon $C G C$. The very large $X$ motif $A l a-t R N A X m_{1} G C, C T C, A A T, G G C, A T T, G A G, G A G, G T C, A$ of 24 nucleotides in $T$. thermophilus is in frame with the anti-codon $G G C$. The $X$ motif $G C, C T G, A A T, C$ which is prefix of $A l a-t R N A X m_{1}$ is in frame with the anti-codon $C G C$.
(iii) 3' region of Ala-tRNAs: The class of $X$ motifs Ala - $t R N A X m_{2}$ $K, S A G, G A G, G T C, W$ is suffix of $A l a-t R N A X m_{1}$. The class of $X$ motifs $R, G A G, G Y C, R$ is suffix of $A l a-t R N A X m_{2}$. Two $X$ motifs are also observed: $A, G G T, C A G, G G$ and $C C, C T C, G G C, T$.

Table 4.6: Identification of $X$ circular code motifs ala-tRNAXm in tRNAs of Alanine (Ala) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos/5' region | AC 3' region |  | \|Organism ID | Start |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TGC 34\|T, CAG, CTG, GG |  |  | E. coli C08004469 | 12 | 20 | 9 |
| TGC 34 T, CAG, CTG, GG |  |  | E. coli C08004522 | 12 | 20 | 9 |
| GGC 34 T, CAG, CTG, GG |  |  | E. coli C08004535 | 12 | 20 | 9 |
| TGC 36 GC, CTG,GTA, T |  |  | P. furiosus At1825 | 15 | 23 | 9 |
| GGC 36 GC, CTG,GTA, T |  |  | P. furiosus At1834 | 15 | 23 | 9 |
| CGC 35 GC, CTG,GAA, GAG, C |  |  | P. furiosus At1833 | 15 | 26 | 12 |
| TGC 36 | GC, GCC, GCC, CT |  | P. furiosus At1825 | 26 | 35 | 10 |
| GGC 36 | GC, GCC, GCC, TT |  | P. furiosus At1834 | 26 | 35 | 10 |
| CGC 35 | GC, GCC, GCC, CT C,GC |  | P. furiosus At1833 | 25 | 37 | 13 |
| GGC 34 | GC, CTC, AAT, $\overline{\text { GGC, }}$ ATT, GAG, GAG, GTC, A |  | T. thermophilus C025943 | 26 | 49 | 24 |
| CGC 34 | GC, CTG, AAT, $\bar{C}$ |  | T. thermophilus C025964 | 26 | 34 | 9 |
| TGC 34 | G, CAG, GAG, GTC, ${ }^{\text {T }}$ |  | E. coli C08004469 | 39 | 49 | 11 |
| TGC 34 | G, CAG, GAG, GTC, T |  | E. coli C08004522 | 39 | 49 | 11 |
| CGC 34 | T, CAG, GAG, GTC, A |  | T. thermophilus C025964 | 39 | 49 | 11 |
| GGC 34 | AA, GAG, GTC, A |  | E. coli C08004535 | 41 | 49 | 9 |
| CGC 35 | G, GAG, GCC, GC |  | P. furiosus At1833 | 43 | 51 | 9 |
| GGC 34 | A, GGT, CAG, GG |  | T. thermophilus C025943 | 44 | 52 | 9 |
| CGC 34 | A, GGT, CAG, GG |  | T. thermophilus C025964 | 44 | 52 | 9 |
| GGC 34 |  | CC, CTC, GGC, T | T. thermophilus C025943 | 62 | 70 | 9 |
| CGC 34 |  | CC, CTC, GGC, $T$ | T. thermophilus C025964 | 62 | 70 | 9 |

Table 4.7: Identification of $X$ circular code motifs arg-tRNAXm in tRNAs of Arginine (Arg) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos/5' region |  | AC 3' region |  | \|Organism ID | Start |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TCG $36 \mid$ G | G, GCC , GGT, GGC , CT |  |  | P. furiosus At1842 | 2 | 13 | 12 |
| GCG 36 | CC, GGT, GGC, CT |  |  | P. furiosus At1850 | 4 | 13 | 10 |
| ACG 35 | CC, GTA, GTT, CAG, CTG, GAT, A |  |  | E. coli C08004532 | 5 | 22 | 18 |
| TCT 35 | CC, GTA, GCC, TA |  |  | P. furiosus At1826 | 5 | 14 | 10 |
| CCT 33 | G, GTA, GCC, TA |  |  | P. furiosus At1852 | 6 | 14 | 9 |
| CCG 36 | G, GTA, GTT, TA |  |  | P. furiosus At1832 | 6 | 14 | 9 |
| CCG 35 | T, CAG, CTG, GAT, A |  |  | E. coli C08004500 | 12 | 22 | 11 |
| ACG 35 | T, CAG, CTG, GAT, A |  |  | E. coli C08004529 | 12 | 22 | 11 |
| ACG 35 | T, CAG, CTG, GAT, A |  |  | T. thermophilus C025939 | 12 | 22 | 11 |
| CCG 35 | T, CAG, CTG, GAT, A |  |  | T. thermophilus C025957 | 12 | 22 | 11 |
| CCT 35 | T, CAG, CAG, GAT, A |  |  | T. thermophilus C025920 | 12 | 22 | 11 |
| TCT 35 | AG, CAG, GAT, A |  |  | P. furiosus At1826 | 14 | 22 | 9 |
| CCG 36 | GC, CAG, GAG, A |  |  | P. furiosus At1832 | 15 | 23 | 9 |
| CCT 33 | GC, CAG, GAT, A |  |  | P. furiosus At1852 | 15 | 23 | 9 |
| TCG 36 | GC, CTG, GAT, GG |  |  | P. furiosus At1842 | 15 | 24 | 10 |
| GCG 36 | GC,CTG, GAT, A |  |  | P. furiosus At1850 | 15 | 23 | 9 |
| CCT 31 | AT, AAC, GAG, C |  |  | E. coli C08004487 | 21 | 29 | 9 |
| ACG 35 | TA, GAG, TAC, T |  |  | E. coli C08004529 | 21 | 29 | 9 |
| ACG 35 | TA, GAG, TAC, T |  |  | E. coli C08004532 | 21 | 29 | 9 |
| TCT 35 |  | AG, GGC , GCC , GGC , CT |  | P. furiosus At1826 | 22 | 34 | 13 |
| TCG 36 |  | G,GGC,GTC, GGC, CT |  | P. furiosus At1842 | 24 | 35 | 12 |
| TCT 35 |  | GC, AAC, GAC, CT |  | E. coli C08004473 | 25 | 34 | 10 |
| CCG 36 |  | A, GAG, AAC , GCC, GCC, CT C, C |  | P. furiosus At1832 | 21 | 37 | 17 |
| CCG 35 |  | GC,GTC, GGC, CT $\overline{C, C}$ |  | T. thermophilus C 025957 | 25 | 36 | 12 |
| CCT 36 |  | AG, GGC, GGC, GGC, CT $\overline{C, C T}$ |  | P. furiosus At1852 | 23 | 38 | 16 |
| CCT 35 |  | G,GGC, TT $\overline{C, C T}$ |  | T. thermophilus C025920 | 29 | 37 | 9 |
| TCT 34 |  | A,GCC, GCC, T $\overline{T C, T} A$ |  | T. thermophilus C025953 | 26 | 37 | 12 |
| TCT 35 |  | $\mathrm{G}, \mathrm{GCC}, \mathrm{T} \overline{T C, T} \mathrm{~A}$ |  | P. furiosus At1826 | 30 | 38 | 9 |
| TCT 35 |  | $\mathrm{G}, \mathrm{ACC}, \mathrm{T} \overline{T C, T} \mathrm{~A}$ |  | E. coli C08004473 | 30 | 38 | 9 |
| TCG 36 |  | G,GCC, T $\overline{T C, G} \mathrm{G}$ |  | P. furiosus At1842 | 31 | 39 | 9 |
| ACG 35 |  | A , GTA , CTC, GGC, ${ }^{\text {AC,G }} \mathrm{AA}, \mathrm{C}$ |  | E. coli C08004529 | 24 | 40 | 17 |
| ACG 35 |  | A, GTA, CTC, GGC, T $\overline{A C, G} A A, C$ |  | E. coli C08004532 | 24 | 40 | 17 |
| ACG 35 |  | T, GAC, $\overline{\overline{A C, G}} \mathrm{G}$ |  | T. thermophilus C025939 | 30 | 38 | 9 |
| GCG 36 |  | G, GAC, CTC, GAG, GTC, C |  | P. furiosus At1850 | 38 | 51 | 14 |
| ACG 35 |  | G, GAT, CAG, CAG , GTC, GG |  | T. thermophilus C025939 | 37 | 51 | 15 |
| CCT 31 |  | TG, CAG,GTT, C |  | E. coli C08004487 | 47 | 55 | 9 |
| CCG 35 |  | A, GCC, GAA, GGT, CAG, A |  | T. thermophilus C025957 | 39 | 52 | 14 |
| TCG 36 |  | A, GCC, GAA, GGT, C |  | P. furiosus At1842 | 40 | 50 | 11 |
| CCT 31 |  |  | A, TTC, CTG, CAG, GG | E. coli C08004487 | 57 | 68 | 12 |
| TCT 35 |  |  | A, ATC, CTG, CAG, GGC , GC | E. coli C08004473 | 59 | 73 | 15 |

Table 4.8: Identification of $X$ circular code motifs asn-tRNAXm in tRNAs of Asparagine (Asn) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos\|5' region |  | AC 3' region | \|Organism ID | Start End Length |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| GTT 34 | GCC, GCC, GTA , GC |  | P. furiosus At1856 | 1 | 11 | 11 |
| GTT 34 | T, GTA, GTT, CAG, T |  | E. coli C08004483 | 6 | 16 | 11 |
| GTT 34 | T, GTA, GTT, CAG, T |  | E. coli C08004537 | 6 | 16 | 11 |
| GTT 34 | T, CAG, CAG, GTA , GAG, CAG, C |  | T. thermophilus C025962 | 12 | 28 | 17 |
| GTT 34 | AG, AAC, GGC, GG |  | E. coli C08004483 | 21 | 30 | 10 |
| GTT 34 | AG, AAC, GGC, GG |  | E. coli C08004537 | 21 | 30 | 10 |
| GTT 34 |  | T A, ACC, GGT, A | T. thermophilus C025962 | 36 | 44 | 9 |
| GTT 34 |  | CC, GGC, GGT, C | P. furiosus At1856 | 40 | 48 | 9 |
| GTT 34 |  |  | P. furiosus At1856 | 63 | 75 | 13 |

Table 4.9: Identification of $X$ circular code motifs asp-tRNAXm in tRNAs of Aspartic (Asp) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos\|5' region | AC 3' region | Organism ID | Start End Length |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| GTC 37 \|G, GGT, GGT, GTA, GCC , C |  | P. furiosus At1869 | 5 | 18 | 14 |
| GTC 35 GT, GGT, GTA, GTT, GGT, TA |  | T. thermophilus C025932 | 7 | 22 | 16 |
| GTC 35 G,GTA,GTT, CAG, T |  | E. coli C08004470 | 6 | 16 | 11 |
| GTC 35 A,GTC,GGT, TA |  | E. coli C08004470 | 14 | 22 | 9 |
| GTC 35 TG, GTT, AAC, AC |  | T. thermophilus C025932 | 17 | 26 | 10 |
| GTC 35 TA, GAA, TAC, CTG, C |  | E. coli C08004470 | 21 | 32 | 12 |
| GTC 35 A, GAA, TAC, CT |  | E. coli C08004470 | 22 | 30 | 9 |
| GTC 35 | G,GAG, ATC, GC | T. thermophilus C025932 | 43 | 51 | 9 |

Table 4.10: Identification of $X$ circular code motifs cys-tRNAXm in tRNAs of Cysteine (Cys) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).


Table 4.11: Identification of $X$ circular code motifs gln-tRNAXm in tRNAs of Glutamine (Gln) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).


Table 4.12: Identification of $X$ circular code motifs glu-tRNAXm intRNAs of Glutamic acid (Glu) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos\|5' region |  | AC 3' region | \|Organism ID | Start End Length |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CTC 35 | CC, GGT, GGT, GTA , GCC , C |  | P. furiosus At1830 | 4 | 18 | 15 |
| TTC 37 | CC, GGT, GGT, GTA, GCC, C |  | P. furiosus At1847 | 4 | 18 | 15 |
| TTC 35 | CC, TTC, GTC, TA |  | E. coli C08004497 | 5 | 14 | 10 |
| TTC 35 | CC, TTC, GTC, TA |  | E. coli C08004534 | 5 | 14 | 10 |
| TTC 34 | CC, ATC, GAC, TA |  | T. thermophilus C025941 | 5 | 14 | 10 |
| CTC 32 | CC, ATC, GTC, TA |  | T. thermophilus C025923 | 5 | 14 | 10 |
| CTC 33 | CC, ATC, GTC, TA |  | T. thermophilus C025942 | 5 | 14 | 10 |
| TTC 35 | TA, GAG, GCC , CAG , GAC , ACC , GCC , CT |  | E. coli C08004497 | 13 | 34 | 22 |
| TTC 35 | TA, GAG, GCC , CAG , GAC , ACC, GCC, CT |  | E. coli C08004534 | 13 | 34 | 22 |
| CTC 33 | TA, GAG, GCC, TA |  | T. thermophilus C025942 | 13 | 22 | 10 |
| TTC 34 | AG, GTC, ACC , GGC , CT |  | T. thermophilus C025941 | 21 | 33 | 13 |
| TTC 34 |  | AA, GCC, GGC, GGC, GG | T. thermophilus C025941 | 37 | 49 | 13 |
| TTC 35 |  | AC, GGC, GGT, AAC, A | E. coli C08004497 | 38 | 49 | 12 |
| TTC 35 |  | AC, GGC, GGT, AAC , A | E. coli C08004534 | 38 | 49 | 12 |
| CTC 32 |  | AG, GCC, GAA , AC | T. thermophilus C025923 | 38 | 47 | 10 |
| CTC 33 |  | AG, GCC, GAG, AC | T. thermophilus C025942 | 39 | 48 | 10 |

### 4.2.3.2 $X$ circular code motifs in Arg-tRNAs (Table 4.7)

(i) 5' region of Arg-tRNAs: The $X$ motif CC,GGT,GGC,CT is found. The large $X$ motif $\operatorname{Arg}-t R N A X m_{1} C C, G T A, G T T, C A G, C T G, G A T, A$ of 18 nucleotides is identified in $E$. coli. The class of $X$ motifs $S, G T A, G Y Y, T A$ is prefix of $\operatorname{Arg}-t R N A X m_{1}$, and the class of $X$
motifs $T, C A G, C W G, G A T, A$ and $A r g-t R N A X m_{2} R S, C W G, G A T, R$ are suffix of $A r g-t R N A X m_{1}$. The $X$ motif $A T, A A C, G A G, C$ and $\operatorname{Arg}-t R N A X m_{2}$ are shifted in frame. The $X$ motif $T A, G A G, T A C, T$ is observed. The class of $X$ motifs $G, G G C, G Y C, G G C, C T$ and the $X$ motif $G C, A A C, G A C, C T$ occur before the anti-codons $T C G$ and $T C T$.
(ii) anti-codon regions of Arg-tRNAs: The large $X$ motif $\operatorname{Arg}-t R N A X m_{3}$ $A, G A G, A A C, G C C, G C C, C T C, C$ of 17 nucleotides in $P$. furiosus is in a different frame than the anti-codon $C C G$. The $X$ motif $G C, G T C, G G C, C T C, C$ which is suffix of $\operatorname{Arg}-t R N A X m_{3}$ is in a different frame than the anti-codon $C C G$. The large $X$ motif $\operatorname{Arg}-t R N A X m_{4}$ $A G, G G C, G G C, G G C, C T C, C T$ of 16 nucleotides in $P$. furiosus is in a different frame than the anti-codon $C C T$. The $X$ motif $G, G G C, T T C, C T$ which is suffix of $\operatorname{Arg}-t R N A X m_{4}$ is in a different frame than the anticodon $C C T$. The $X$ motif $\operatorname{Arg}-t R N A X m_{5} A, G C C, G C C, T C T, T A$ is in a different frame than the anti-codon $T C T$. The class of $X$ motifs $G, R C C, T C T, T A$ which is suffix of $\operatorname{Arg}-t R N A X m_{5}$ is in a different frame than the anti-codon $T C T$. The $X$ motif $G, G C C, T T C, G G$ is in a different frame than the anti-codon $T C G$. The large $X$ motif $\operatorname{Arg}-t R N A X m_{6}$ $A, G T A, C T C, G G C, T A C, G A A, C$ of 17 nucleotides in $E$. coli is in a different frame than the anti-codon $A C G$. The $X$ motif $T, G A C, T A C, G G$ is in a different frame than the anti-codon $A C G$. The class of $X$ motifs $G, G A Y, C W S, S A G, G T C, S$ is in frame with the anti-codons $A C G$ and $G C G$.
(iii) 3' region of Arg-tRNAs: The $X$ motifs $A, G C C, G A A, G G T, C A G, A$ and the class of $X$ motifs $A, W T C, C T G, C A G, G G$ are observed.

### 4.2.3.3 $X$ circular code motifs in Asn-tRNAs (Table 4.8)

(i) 5' region of Asn-tRNAs: The $X$ motif Asn - tRNAXm $m_{1}$ $G C C, G C C, G T A, G C$ is observed. The $X$ motif Asn - tRNAXm $m_{2}$ $T, G T A, G T T, C A G, T$ and $A s n-t R N A X m_{1}$ are shifted in frame. The large $X$ motif $A s n-t R N A X m_{3} T, C A G, C A G, G T \underline{A}, G A G, \underline{C A G}, C$ of 17 nucleotides in $T$. thermophilus and $A s n-t R N A X m_{2}$ are shifted in frame. The $X$ motif $\underline{A G, A} A C, G \underline{G C}, G G$ is shifted by +2 nucleotides from $A s n-t R N A X m_{3}$ (underlined nucleotides).
(ii) anti-codon regions of Asn-tRNAs: The $X$ motif $A s n-t R N A X m_{4}$ $T A, A C C, G G T, A$ is in a different frame than the anti-codon GTT.
(iii) 3' region of Asn-tRNAs: The $X$ motif CC,GGC,GGT,C and $A s n-t R N A X m_{4}$ are shifted in frame. The $X$ motif $G, G G C, G G C, G G C, G C C i s o b s e r v e d$.

### 4.2.3.4 $X$ circular code motifs in Asp-tRNAs (Table 4.9)

(i) 5' region of Asp-tRNAs: The $X$ motif Asp - tRNAX $m_{1}$ $G, G G T, G G T, G T A, G C C, C$ is observed. The large $X$ motif Asp - tRNAXm 2 GT, GGT, GTA, GTT, GGT,TA of 16 nucleotides in T. thermophilus and $A s p-t R N A X m_{1}$ are shifted in frame. The $X$ motif $A s p-t R N A X m_{3} G, G T A, G T T, C A G, T$ is suffix of $A s p-t R N A X m_{2}$. The $X$ motif $A s p-t R N A X m_{4} A, G T C, G G T, T A$ is shifted by +1 nucleotide from $A s p-t R N A X m_{3}$. The $X$ motif $T G, G T T, A A C, A C$ is shifted by +2 nucleotides from $A s p-t R N A X m_{4}$. The $X$ motif $T A, G A A, T A C, C T G, C$ is observed.
(ii) 3' region of Asp-tRNAs: The $X$ motif $G, G A G, A T C, G C$ is observed.

### 4.2.3.5 $X$ circular code motifs in Cys-tRNAs (Table 4.10)

(i) 5' region of Cys-tRNAs: The $X$ motif Cys - tRNAXm $m_{1}$ $G G C, G C C, G T A, G C C, A A$ is observed. The $X$ motif $G C, G T T, A A C, A A$ is suffix of Cys - $t R N A X m_{1}$. The $X$ motif $\underline{A, G G C, C A G}, G C$ is shifted by +1 nucleotide from $T A, G \underline{A G, G C C, A}$.
(ii) anti-codon regions of Cys-tRNAs: The $X$ motif Cys - tRNAXm $m_{2}$ $A, G G T, C T G, C A$ is in a different frame than the anti-codon $G C A$. The $X$ motif Cys - tRNAXm $m_{3} A, C T G, C A G, A T C, C$ and $C y s-t R N A X m_{2}$ are shifted in frame, thus $C y s-t R N A X m_{3}$ is in a different frame than the anti-codon $G C A$. The $X$ motifs $A A, A A C, C T C, C A$ and $C y s-t R N A X m_{3}$ are shifted in frame, thus they are in a different frame than the anti-codon $G C A$.
(iii) 3' region of Cys-tRNAs: Two $X$ motifs $A, T T C, G C C, G G T, T$ and $G, G C C, G G C, G C C, T$ are observed.

### 4.2.3.6 $X$ circular code motifs in Gln-tRNAs (Table 4.11)

(i) 5' region of Gln-tRNAs: The class of $X$ motifs $G l n-t R N A X m_{1}$ $G, G G Y, R T C, G Y C, W A$ is identified. The $X$ motif $\underline{G T}, G \underline{G T}, G \underline{T A}, G C$ is shifted by +1 nucleotide from the class of $X$ motifs $G, G G Y, \underline{G T} C, \underline{G T C}, \underline{T A}$ belonging to $G l n-t R N A X m_{1}$. The $X$ motif $A A, G G C, A C C, G G T, T T$ occurs before the anti-codon.
(ii) anti-codon regions of Gln-tRNAs: The $X$ motif $G, A T T, C T G, A T T, C$ is in frame with the anti-codon $C T G$. The $X$ motif $G l n-t R N A X m_{2}$ $T T, G A T, A C C, G G C, A T T, C$ is in a different frame than the anti-codon $T T G$.
(iii) 3' region of Gln-tRNAs: The $X$ motif $C C, G C C, G G T, G G T, G G T, T$ and $G l n-t R N A X m_{2}$ are shifted in frame. Two classes of $X$ motifs $C C, S W G, G T T, C$ and $A, A T C, C W S, G T A, C$ are observed.

### 4.2.3.7 $X$ circular code motifs in Glu-tRNAs (Table 4.12)

(i) 5' region of Glu-tRNAs: The $X$ motif Glu - $t R N A X m_{1}$ $C \underline{C}, G G \underline{T}, G \underline{G} T, G \underline{T A}, G C C, C$ is observed. The class of $X$ motifs $\underline{C} C, W \underline{T} C, \underline{G} W C, \underline{T A}$ is shifted by +2 nucleotides from $G l u-t R N A X m_{1}$. A very large $X$ motif $G l u-t R N A X m_{2}$ $T A, G A G, G C C, C A G, G A C, A C C, G C C, C T$ of 22 nucleotide in $E$. coli and Glu - tRNAXm $m_{1}$ are shifted in frame. The $X$ motif $T A, G A G, G C C, T A$ is prefix of $G l u-t R N A X m_{2}$ and the $X$ motif $A G, G T C, A C C, G G C, C T$ is suffix of $G l u-t R N A X m_{2}$.
(ii) 3' region of Glu-tRNAs: The $X$ motifs $A A, G C C, G G C, G G C, G G$ and $G l u-t R N A X m_{3} A C, \underline{G G C}, G \underline{G} T, A \underline{A C}, A$ occur after (3') the anti-codon $T T C$. The class of $X$ motifs $A \underline{G}, G C C, \underline{G} A R, \underline{A C}$ is shifted by $+2 \mathrm{nu}-$ cleotides from $G l u-t R N A X m_{3}$.

### 4.2.3.8 $X$ circular code motifs in Gly-tRNAs (Table 4.13)

(i) 5' region of Gly-tRNAs: Several classes of $X$ motifs are identified: $G C, G G T, G G T, A, G l y-t R N A X m_{1} G, G G C, R T M, G T W, Y A$, $T G, G T A, G T C, T A$ suffix of Gly - tRNAXm $m_{1}, W Y, A A T, G G Y, W$, $G C, C T G, G T C, T A, \quad G, G T A, G A R, C$ and $A K, W A C, S W S, A$. The

Table 4.13: Identification of $X$ circular code motifs gly-tRNAXm in tRNAs of Glycine (Gly) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).


Table 4.14: Identification of $X$ circular code motifs his-tRNAXm in tRNAs of Histidine (His) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos $5^{\prime}$ ' region | AC 3' region | \|Organism ID | Start End Length |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| GTG $36 \mid \mathrm{G}, \mathrm{GGT}, \mathrm{GGT}, \mathrm{GTA}, \mathrm{GCC}, \mathrm{T}$ |  | P. furiosus At1843 | 5 | 18 | 14 |
| GTG 35 T, CAG, CTG,GTT,A |  | T. thermophilus C 025927 | 12 | 22 | 11 |
| GTG 36 GC, CTG,GTT,A |  | $P$. furiosus At1843 | 15 | 23 | 9 |
| GTG 34 TG, GTA, GAG, C |  | E. coli C08004501 | 17 | 25 | 9 |
| GTG 34 A, GCC, CTG, GAT, T |  | E. coli C08004501 | 23 | 33 | 11 |
| GTG 34 | A, TTC, CAG, TT | E. coli C08004501 | 37 | 45 | 9 |
| GTG 34 | A, GTT, GTC, GT | E. coli C08004501 | 42 | 50 | 9 |
| GTG 36 | CC, CTG, GCC , C | $P$ furiosus At1843 | 43 | 51 | 9 |

two $X$ motifs $G l y-t R N A X m_{2} A G, G A C, G C C, G G C, C T$ and $G l y-$ $t R N A X m_{3} G C, A T C, G G C, C T$ occur before the anti-codon TCC.
(ii) anti-codon regions of Gly-tRNAs: The two $X$ motifs $A G, G A C, G C C, G G C, C T C, C$ whose prefix is Gly - $t R N A X m_{2}$ and $G C, A T C, G G C, T T C, C$ whose prefix is Gly - $t R N A X m_{3}$ are in a different frame than the anti-codon $C C C$. The $X$ motif

Table 4.15: Identification of $X$ circular code motifs ile-tRNAXm in tRNAs of Isoleucine (IIe) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos\|5' region | AC 3' region | \|Organism ID | Start End Length |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| GAT 35 G, GGC, GAT, TA |  | T. thermophilus C025963 | 1 | 9 | 9 |
| GAT 35 T, CAG, CTG, GTT, A |  | T. thermophilus C025963 | 12 | 22 | 11 |
| GAT 36 GC, CTG, GTC, A |  | P. furiosus At1831 | 15 | 23 | 9 |
| GAT 35 A, GGT, GGT, TA |  | E. coli C08004468 | 14 | 22 | 9 |
| GAT 35 A, GGT, GGT, TA |  | E. coli C08004521 | 14 | 22 | 9 |
| GAT 35 | AG, GGT, GAG, GTC, GGT, GGT, T | E. coli C08004468 | 39 | 56 | 18 |
| GAT 35 | AG, GGT, GAG, GTC, GGT, GGT, T | E. coli C08004521 | 39 | 56 | 18 |
| GAT 35 | GT, GAG, GTC, GGT, GGT, T | T. thermophilus C025963 | 42 | 56 | 15 |
| GAT 36 | G,TTC, GAA, GCC, C | P. furiosus At1831 | 55 | 65 | 11 |
| GAT 35 | CC, ACC, ATC, GCC, CA | T. thermophilus C025963 | 62 | 74 | 13 |
| GAT 35 | T, CAG, GCC, TAC | E. coli C08004468 | 66 | 75 | 10 |

Table 4.16: Identification of $X$ circular code motifs leu-tRNAXm in tRNAs of Leucine (Leu) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos 5 ', region |  | AC 3' region | \|Organism ID | Start | End | gth |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CAG 35 | GG , GAA , GGT , GGC , GG |  | E. coli C08004502 | 1 | 13 | 13 |
| CAG 35 | GC, GAA, GGT, GGC, GG |  | E. coli C08004516 | 1 | 13 | 13 |
| CAG 35 | GC, GAA , GGt, GGC, GG |  | E. coli C08004517 | 1 | 13 | 13 |
| TAG 35 | GC, GAG, GAT, GGC , GG |  | T. thermophilus C025928 | 1 | 13 | 13 |
| TAA 34 | G,GGT, GGC, GG |  | T. thermophilus C025929 | 5 | 13 | 9 |
| CAG 35 | G,GGT, GGC, GG |  | T. thermophilus C025947 | 5 | 13 | 9 |
| CAA 35 | G,GGT, GGC,GG |  | T. thermophilus C025954 | 5 | 13 | 9 |
| GAG 35 | A,GGT, GGt, GG |  | E. coli C08004524 | 5 | 13 | 9 |
| GAG 35 | G,GGT,GGT,GG |  | T. thermophilus C025921 | 5 | 13 | 9 |
| TAA 35 | G,GAT, GGT, GG |  | E. coli C08004541 | 5 | 13 | 9 |
| CAG 37 | G,GTT, GCC, GAG, C |  | P. furiosus At1867 | 6 | 16 | 11 |
| TAG 37 | G,GTt, GCC, GAG, C |  | P. furiosus At1838 | 6 | 16 | 11 |
| CAA 37 | G,GTt, GCC, GAG, C |  | P. furiosus At1848 | 6 | 16 | 11 |
| GAG 37 | G,GTT,GCC, GAG, C |  | P. furiosus At1853 | 6 | 16 | 11 |
| CAA 35 | GT, GGC, GAA , ATC, GGT, A |  | E. coli C08004515 | 7 | 21 | 15 |
| TAG 35 | GT, GGC, GAA , ATT, GGT, A |  | E. coli C08004548 | 7 | 21 | 15 |
| GAG 35 | TG, GAA, CTG, GTA, GAC , AC |  | T. thermophilus C025921 | 11 | 26 | 16 |
| CAG 37 | GC, CTG,GTC, AA |  | P. furiosus At1867 | 15 | 24 | 10 |
| TAG 37 | GC, CTG, GTC, AA |  | P. furiosus At1838 | 15 | 24 | 10 |
| CAA 37 | GC, CTG, GTC, AA |  | P. furiosus At1848 | 15 | 24 | 10 |
| GAG 37 | GC, CTG, GTC, AA |  | P. furiosus At1853 | 15 | 24 | 10 |
| TAA 37 | GC, CTG, GCC, AA |  | P. furiosus At1862 | 15 | 24 | 10 |
| CAG 35 | TG,GTA,GAC,GC |  | E. coli C08004502 | 17 | 26 | 10 |
| CAG 35 | TG, GTA, GAC, GC |  | E. coli C08004516 | 17 | 26 | 10 |
| CAG 35 | TG, GTA, GAC, GC |  | E. coli C08004517 | 17 | 26 | 10 |
| GAG 35 | TG, GTA, GAC, AC |  | E. coli C08004524 | 17 | 26 | 10 |
| TAG 35 | TG, GTA, GAC, GC |  | E. coli C08004548 | 17 | 26 | 10 |
| CAG 35 | tG, GTA, GAC, GC |  | T. thermophilus C025947 | 17 | 26 | 10 |
| TAA 34 | G,GTA, GAC, GC |  | T. thermophilus C025929 | 17 | 25 | 9 |
| CAA 35 | G,GTA, GAC, GC |  | E. coli C08004515 | 18 | 26 | 9 |
| TAA 35 | G,GTA, GAC, AC |  | E. coli C08004541 | 18 | 26 | 9 |
| TAG 35 | G,GTA,GAC, GC |  | T. thermophilus C025928 | 18 | 26 | 9 |
| CAA 35 | G, GTA, GAC, GC |  | T. thermophilus C025954 | 18 | 26 | 9 |
| GAG 35 | AC, GCC, ATC, TT |  | T. thermophilus C025921 | 25 | 34 | 10 |
| TAG 35 | AC, CAG, ATT, |  | E. coli C08004548 | 27 | 36 | 10 |
| CAG 35 | TG, ATt, | CAG, GGT, CA | T. thermophilus C025947 | 30 | 42 | 13 |
| CAG 37 | G,ATt, | CAG, GGT, C | P. furiosus At1867 | 33 | 43 | 11 |
| GAG 37 | G,ATT, | GAG, GGT, C | P. furiosus At1853 | 33 | 43 | 11 |
| TAG 35 |  | $\overline{\text { AG, }}$ GTT, CTG, GC | E. coli C08004548 | 36 | 45 | 10 |
| CAA 35 |  | AA, AAT, CTG, CTG , T | T. thermophilus C025954 | 36 | 47 | 12 |
| CAA 35 |  | AA, ATC, AAC, C | E. coli C08004515 | 37 | 45 | 9 |
| CAA 35 |  | - A, ACC , GTA, GAA, AT | E. coli C08004515 | 42 | 53 | 12 |
| TAA 35 |  | CC, CTC, GGC, GTT, C | E. coli C08004541 | 41 | 52 | 12 |
| TAG 35 |  | T,GGC, GCC, GC | E. coli C08004548 | 42 | 50 | 9 |
| GAG 35 |  | $\mathrm{G}, \mathrm{GGT}, \mathrm{GGT}, \mathrm{GCC}, \mathrm{C}$ | T. thermophilus C 025921 | 39 | 49 | 11 |
| TAA 37 |  | CC, GGT, GCC, GTA, GG | P. furiosus At1862 | 43 | 55 | 13 |
| CAA 35 |  | GT, GCC, GGT, T | E. coli C08004515 | 56 | 64 | 9 |
| TAG 35 |  | GC, GAG, TTC , AA | E. coli C08004548 | 58 | 67 | 10 |
| TAG 35 |  | GT, CTC,GCC, T | E. coli C08004548 | 68 | 76 | 9 |
| CAA 35 |  | G,GCC, TTC, GGC, ACC | E. coli C08004515 | 72 | 84 | 13 |
| TAG 35 |  | G,CTC,CTC,GC | T. thermophilus C025928 | 73 | 81 | 9 |

$A G, G A C, G C C, A C C, C T G, C$ is in a different frame than the anti-codon $G C C$. The class of $X$ motifs $R, G C C, T T C, C A$ is in a different frame than the anti-codon $T C C$.
(iii) 3' region of Gly-tRNAs: The class of $X$ motifs $A A, G C C, G R S, G R Y, C$ oc-

Table 4.17: Identification of $X$ circular code motifs lys-tRNAXm in tRNAs of Lysine (Lys) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).


Table 4.18: Identification of $X$ circular code motifs met-tRNAXm intRNAs of Methionine (Met) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos\|5 | 5' region | AC 3' region | \|Organism ID | Start |  | th |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CAT 35 | GGC , TAC , GTA, GC |  | E. coli C08004547 | 1 | 11 | 11 |
| CAT 35 | GGC, GGC, GTA, GC |  | T. thermophilus C025934 | 1 | 11 | 11 |
| CAT 35 | TG, GAG, CAG, C |  | E. coli C08004492 | 8 | 16 | 9 |
| CAT 35 | TG, GAG, CAG, C |  | E. coli C08004525 | 8 | 16 | 9 |
| CAT 35 | TG, GAG, CAG, C |  | T. thermophilus C025937 | 8 | 16 | 9 |
| CAT 34 | AG, CTC, AAC, GGT, CAG, A |  | T. thermophilus C025955 | 9 | 23 | 15 |
| CAT 35 | A, GGT, GGT, CAG, A |  | T. thermophilus C025934 | 14 | 24 | 11 |
| CAT 35 | A, GTT, GGT, TA |  | E. coli C08004547 | 14 | 22 | 9 |
| CAT 36 | GC, CTG, GTC, AA |  | P. furiosus At1857 | 15 | 24 | 10 |
| CAT 36 | GC, CTG, GTC, A |  | P. furiosus At1855 | 15 | 23 | 9 |
| CAT 35 | GC, CTG, GTA, GC |  | E. coli C08004492 | 15 | 24 | 10 |
| CAT 35 | GC, CTG, GTA, GC |  | E. coli C08004525 | 15 | 24 | 10 |
| CAT 35 | GC, CTG, GTA, GC |  | T. thermophilus C025937 | 15 | 24 | 10 |
| CAT 34 | TA, GAG, CAG, GC |  | E. coli C08004495 | 20 | 29 | 10 |
| CAT 34 | TA, GAG, CAG, GC |  | E. coli C08004533 | 20 | 29 | 10 |
| CAT 35 | AG, CTC, GTC, GG |  | E. coli C08004492 | 22 | 31 | 10 |
| CAT 35 | AG, CTC, GTC, GG |  | E. coli C08004525 | 22 | 31 | 10 |
| CAT 35 | AG, CTC, GTC, GG |  | T. thermophilus C025937 | 22 | 31 | 10 |
| CAT 35 |  | AT, AAT, GAT, GG | E. coli C08004547 | 36 | 45 | 10 |
| CAT 34 |  | - T $A, A C C, G G T, A$ | T. thermophilus C025955 | 36 | 44 | 9 |
| CAT 35 |  | CC, GAA, GGT, C | E. coli C08004492 | 41 | 49 | 9 |
| CAT 35 |  | CC, GAA, GGT, C | T. thermophilus C025937 | 41 | 49 | 9 |
| CAT 35 |  | CC, GAA, GAT, C | E. coli C08004525 | 41 | 49 | 9 |
| CAT 35 |  | GT, GGT, GTC, GT | T. thermophilus C025934 | 42 | 51 | 10 |
| CAT 35 |  | AG, GTC, GTC, GGT, T | E. coli C08004492 | 45 | 56 | 12 |
| CAT 35 |  | AG, ATC, GTC, GGT, T | E. coli C08004525 | 45 | 56 | 12 |
| CAT 34 |  | TG, CAG, GTT, C | T. thermophilus C025955 | 48 | 56 | 9 |
| CAT 36 |  | CC, GAG, GTT, CA | P. furiosus At1846 | 50 | 59 | 10 |
| CAT 34 |  | G,CTG, GTT, CA | E. coli C08004495 | 49 | 57 | 9 |
| CAT 34 |  | G, CTG, GTT, CA | E. coli C08004533 | 49 | 57 | 9 |
| CAT 36 |  | G,TTC, GAA, GCC, C | P. furiosus At1857 | 55 | 65 | 11 |
| CAT 34 |  | AA, GTC, CAG, CAG, GG | E. coli C08004495 | 57 | 69 | 13 |
| CAT 34 |  | AA, GTC, CAG, CA | E. coli C08004533 | 57 | 66 | 10 |
| CAT 36 |  | AA, ATC, CTC, GGC, C | P. furiosus At1846 | 59 | 70 | 12 |
| CAT 34 |  | A, ATC, CTG, CA | T. thermophilus C025955 | 58 | 66 | 9 |
| CAT 35 |  | CC, GTC, GTA, GCC | E. coli C08004547 | 63 | 73 | 11 |
| CAT 35 |  | $\mathrm{CC}, \mathrm{ACC}, \mathrm{GCC}, \mathrm{GCC}, \mathrm{ACC}$ | T. thermophilus C025934 | 63 | 76 | 14 |

curs after the anti-codons $C C C$ and $T C C$. Several $X$ motifs are also found:
$T A, T A C, G A G, G G T, T, \quad T G, G A G, A C C, C, \quad G C, G A G, T T C, G A G, T$, $G T, C T C, G T T, T, C C, T T C, G C C, C$ and $G, G C C, A C C, G C$ where no obvious relation could have been identified between them so far.

Table 4.19: Identification of $X$ circular code motifs phe-tRNAXm in tRNAs of Phenylalanine (Phe) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos\|5' region | AC 3' region | Organism ID | Start End Length |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| GAA $34 \mid$ GCC, GAG, GTA, GC |  | T. thermophilus C025933 | 1 | 11 | 11 |
| GAA 34 | TG, GTA, GAG, CA | T. thermophilus C025933 | 17 | 26 | 10 |
| GAA 34 | G, GTA, GAG, CAG, GG | E. coli C08004519 | 18 | 29 | 12 |
| GAA 34 | G, ATT, GAA, AAT, C | E. coli C08004519 | 30 | 40 | 11 |
| GAA 35 | G, GGT, GTC, GG | P. furiosus At1845 | 43 | 51 | 9 |
| GAA 34 | GT, GTC, GGC, GGT, T | T. thermophilus C025933 | 44 | 55 | 12 |
| GAA 34 | CC, CTC, GGC , ACC | T. thermophilus C025933 | 65 | 75 | 11 |

### 4.2.3.9 $X$ CIRCULAR CODE motifs in His-TRNAs (TABLE 4.14)

(i) 5' region of His-tRNAs: Several classes of $X$ motifs are identified: $G, G G T, G G T, G T A, G C C, T, H i s-t R N A X m_{1} R S, C T G, G T T, A$, $T G, G T A, G A G, C$ shifted in frame with His $-t R N A X m_{1}$ and $A, G C C, C T G, G A T, T$.
(ii) 3' region of His-tRNAs: The $X$ motif $\mathrm{His}^{\prime}-t R N A X m_{2} \underline{A, G T T}, G T C, G T$ is shifted by +1 nucleotide from $A, T T C, C A G, T T$. The $X$ motif $C C, C T G, G C C, C$ is shifted in frame with His $-t R N A X m_{2}$.
4.2.3.10 $X$ circular code motifs in Ile-tRNAs (Table 4.15)
(i) 5' region of Ile-tRNAs: Several $X$ motifs are identified: $G, G G C, G A T, T A$ and $I l e-t R N A X m_{1} R S, C \underline{T G, G T Y}, A$. The $X$ motif $A, G G \underline{T, G G T}, T \underline{A}$ is shifted by +1 nucleotide from $I l e-t R N A X m_{1}$.
(ii) 3 ' region of Ile-tRNAs: A large $X$ motif Ile - tRNAXm $m_{2}$ $A G, G G T, G A G, G T C, G G T, G G T, T$ of 18 nucleotides is identified in E. coli. A suffix of Ile $-t \overline{R N A X} m_{2}$ of 15 nucleotides is found in T. thermophilus. The $X$ motif $\underline{G, T T C}, G A A, G C C, C$ is shifted by +1 nucleotide from $I l e-t R N A X m_{2}$. The $X$ motifs are observed: $C C, A A C, A T C, G C C, C A$ and $T, C A G, G C C, T A C$.

### 4.2.3.11 $X$ circular code motifs in Leu-tRNAs (Table 4.16)

(i) 5' region of Leu-tRNAs: Several classes of $X$ motifs are shifted in frame, in series: $G C, G A R, G R T, G G C, G G, R, G R T, G G Y, G G$, $G, G T T, G C C, G A G, C, G T, G G C, G A A, A T Y, G G T, A$, a large $X$ motif Leu - tRNAXm $\quad T G, G A A, C T G, G T A, G A C, A C$ of 16 nucleotides in $T$. thermophilus, $G C, C T G, G Y C, A A, G, G T A, G A C, R C$ and $A C, G C C, A T C, T T$ which occurs before the anti-codon $G A G$.
(ii) anti-codon regions of Leu-tRNAs: The $X$ motif Leu - tRNAXm $m_{2}$ $A C, C A G, A T T, T A$ is in frame with the anti-codon $T A G$. The class of $X$ motifs $G, A T T, S A G, G G T, C$ in frame with Leu - tRNAXm $m_{2}$ except with its suffix $T A$ of the anti-codon $T A G$, is in frame with both the anti-codons $C A G$ and $G A G$. The $X$ motifs $A G, G T T, C T G, G C$, $A A, A A T, C T G, C T G, T$ and $L e u-t R N A X m_{3} A A, A T C, \underline{A A C, C}$ are in frame with the anti-codons $T A G, C A A$ and $C A A$, respectively.
(iii) 3' region of Leu-tRNAs: The $X$ motif Leu $-t R N A X m_{4}$ $\underline{A, A C C}, G T A, G A A, A T$ is shifted by +2 nucleotides from Leu - $t R N A X m_{3}$. Several classes of $X$ motifs are shifted in frame: Leu - tRNAXm4, CC, CTC, GGC, GTT, C, T, GGC, GCC,GC and $S Y, G G T, G C C, S$. The $X$ motif $\underline{G C}, G A G, T T C, A A$ is shifted by +1 nucleotide from $G T, \underline{G C C}, G G T, T$. The $X$ motifs $G T, C T C, G C C, T$ and $G, G C C, T T C, G G C, A C C$ are shifted in frame.

### 4.2.3.12 $X$ circular code motifs in Lys-tRNAs (Table 4.17)

(i) 5' region of Lys-tRNAs: The $X$ motif His-tRNAX $m_{1} \underline{T, C A G}, \underline{C T} G, G C$ is shifted by +1 nucleotide from $A G, C \underline{T C, A A C, T}$. The $X$ motifs His - tRNAXm $, ~ G C, C T G, G T T, A, A A, C T G, G T A, G A G, C A$ and $H i s-t R N A X m_{2} T G, G T A, G A G, C \underline{A G, T T}$ are shifted in frame.
(ii) anti-codon regions of Lys-tRNAs: The class of $X$ motifs $\underline{A, R Y Y}, G A C, T T$ which is in frame with the anti-codon $T T T$, is shifted by +1 nucleotide from $H i s-t R N A X m_{2}$. The $X$ motif $\mathrm{His}-t R N A X m_{3} T A, A T C, G G T, G G$ in T. thermophilus C 025946 is in frame with the anti-codon $C T T$.
(iii) 3' region of Lys-tRNAs: Interestingly, the $X$ motifs $T A, A T C, G G T, A$ in T. thermophilus C025922 which is identical to His - tRNAXm (except its last letter) and $T A, A T C, A A T, T$ occur after the anti-codon $T T T$ and are not involved in the anti-codon of Lys- tRNAs. Several classes of $X$ motifs are observed: $T G, G G T, T A C, A, C C, G G T, G G T, C, R, C A G, G T Y, S$ and $A, A T C, C T G, C A$.

### 4.2.3.13 $X$ circular code motifs in Met-tRNAs (Table 4.18)

(i) 5' region of Met-tRNAs: Several classes of $X$ motifs are shifted in frame: $G G C, K R C, G T A, G C, T G, G A G, C A G, C$ and $M e t-t R N A X m_{1}$
$A G, C T C, A A C, G G T, C A G, A$. The $X$ motif Met $-t R N A X m_{2}$ $A, G G T, G G T, C A G, A$ is suffix of Met $-t R N A X m_{1}$. The $X$ motif Met $-t R N A X m_{3} A, G T T, G G T, T A$ is prefix of Met $-t R N A X m_{2}$. The class of $X$ motifs Met $-t R N A X m_{4} G C, C T G, G T M, R$ is shifted by +2 nucleotides from Met - $t R N A X m_{3} A, G T T, G G T, T A$. The class of $X$ motifs Met - tRNAXm $4, T A, G A G, C A G, G C$ and $A G, C T C, G T C, G G$ are shifted in frame.
(ii) anti-codon regions of Met-tRNAs: The $X$ motif $A T, A A T, G A T, G C$ is in frame with the anti-codon $C A T$. The $X$ motif $T A, A C C, G G T, A$ is in a different frame than the anti-codon $C A T$.
(iii) 3' region of Met-tRNAs: Several classes of $X$ motifs are shifted in series: $C C, G G A, G R T, C, G T, G G T, G T C, G T$, Met $-t R N A X m_{5}$ $A G, R T C, G T C, \underline{G G T, T}$, Met $-t R N A X m_{6} S, S W \underline{G, \underline{G T T}, C}$ shifted by +2 nucleotides from Met $-t R N A X m_{5}, G, T T C, G A A, G C C, C$ shifted by +1 nucleotide from $\mathrm{Met}-t R N A X m_{6}, A A, G T C, C A G, C A G, G G$, $A, A T C, C T S, S, C C, G T C, G T A, G C C$ and $C C, A C C, G C C, G C C, A C C$.

Table 4.20: Identification of $X$ circular code motifs pro-tRNAXm in tRNAs of Proline (Pro) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).


Table 4.21: Identification of $X$ circular code motifs sec-tRNAXm in tRNAs of selenocysteine (Sec) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).


Table 4.22: Identification of $X$ circular code motifs ser-tRNAXm in tRNAs of Serine (Ser) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).


### 4.2.3.14 $X$ circular code motifs in Phe-tRNAs (Table 4.19)

(i) 5' region of Phe-tRNAs: Two $X$ motifs $G C C, G A G, G T A, G C$ and $G, G T A, G A G, C A$ are observed.
(ii) anti-codon regions of Phe-tRNAs: The $X$ motif $G, A T T, G A A, A A T, C$ is in frame with the anti-codon $G A A$.
(iii) 3' region of Phe-tRNAs: The $X$ motifs $G, G G T, G T C, G G$ and

Table 4.23: Identification of $X$ circular code motifs thr-tRNAXm in tRNAs of Threonine (Thr) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).


Table 4.24: Identification of $X$ circular code motifs trp-tRNAXm in tRNAs of Tryptophan (Trp) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

$G T, G T C, G G C, G G T, T$ are shifted in frame. The $X$ motif $C C, C T C, G G C, A C C$ is observed.

### 4.2.3.15 $X$ circular code motifs in Pro-tRNAs (Table 4.20)

(i) 5' region of Pro-tRNAs: Several classes of $X$ motifs are shifted in series: $\quad S, G S C, G W R, K$, Pro $-t R N A X m_{1} G, C A G, \underline{G C C}, G \underline{G} T, A$,
 $C C, A T C, C T G, C$ and $G C, A C C, G T C, A T$.

Table 4.25: Identification of $X$ circular code motifs tyr-tRNAXm in tRNAs of Tyrosine (Tyr) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

| AC pos\|5', region | AC 3' region | \|Organism ID | Start | End | th |
| :---: | :---: | :---: | :---: | :---: | :---: |
| GTA $36 \mid$, GTA, GCC, TA |  | P. furiosus At1836 | 6 | 14 | 9 |
| GTA 36 | GC, CTG, GTA , GT | P. furiosus At1836 | 15 | 24 | 10 |
| GTA 36 | GT, GGC, GGC, GG | P. furiosus At1836 | 23 | 32 | 10 |
| GTA 35 | G, GAG, CAG, AC | E. coli C08004508 | 25 | 33 | 9 |
| GTA 35 | G,GAG, CAG, AC | E. coli C08004542 | 25 | 33 | 9 |
| GTA 35 | G,GAC, GGT, CT G,TA | T. thermophilus C025959 | 26 | 37 | 12 |
| GTA 35 | T, GTA, AAT, CTG, C | E. coli C08004508 | 34 | 44 | 11 |
| GTA 35 | T, GTA, AAT, CTG, C | E. coli C08004542 | 34 | 44 | 11 |
| GTA 35 | AA , ACC , GTT, GGC , GTA, T | T. thermophilus C025959 | 38 | 52 | 15 |
| GTA 35 | T, GCC, GTC , ATC, GAC , TTC , GAA , GGT , T | E. coli C08004542 | 42 | 64 | 23 |
| GTA 35 | T, GCC, GTC, AC | E. coli C08004508 | 42 | 50 | 9 |
| GTA 35 | A, GAC , TTC, GAA , GGT , T | E. coli C08004508 | 51 | 64 | 14 |
| GTA 35 | AT, GCC, TTC, GC | T. thermophilus C025959 | 51 | 60 | 10 |

Table 4.26: Identification of $X$ circular code motifs val-tRNAXm in tRNAs of Valine (Val) in prokaryotes (bacteria E. coli and T. thermophilus, archaea P. furiosus).

(ii) anti-codon regions of Pro-tRNAs: The class of $X$ motifs Pro $-t R N A X m_{2}$ $A M, Y T S, G T T, Y$ is in frame with the anti-codons $C G G$ and $T G G$. The class of $X$ motifs $S, G G C, Y T R, G G$ is in a different frame than the anticodons $C G G$ and $G G G$. The class of $X$ motifs $G, G G Y, G Y C, G G$ is in a different frame than the anti-codon $G G G$. The class of $X$ motifs Pro - $t R N A X m_{3} G, G A S, S A R, G G$ is in frame with the anti-codons $C G G$ and $T G G$. The two classes of $X$ motifs Pro $-t R N A X m_{2}$ and Pro - $t R N A X m_{3}$ may derive from an ancestral class of $X$ motifs constructed by the concatenation of Pro $-t R N A X m_{2}$ and Pro $-t R N A X m_{3}$ $A M, Y T S, G T T, Y A G, G A S, S A R, G G$ of 19 nucleotides. Indeed, $C A G$
belongs to $X$ (Equation 2.2). Then, the nucleotide $A$ in the middle site of $C A G$ has mutated to $G$ for building the anti-codon $C G G$.
(iii) 3' region of Pro-tRNAs: The class of $X$ motifs $\underline{G}, S W \underline{G}, G T T, C A$ is shifted by +2 nucleotides from the $X$ motif $G, G T C, \underline{G} T C, \underline{G G T, T}$. Several classes of $X$ motifs are observed: $A, A T C, C W S, T, C C, G G C, S K C, C$ and $T, C T C, G C C, G A C$.

### 4.2.3.16 $X$ circular code motifs in SeC-tRNAs (Table 4.21)

(i) 5' region of SeC-tRNAs: Two $X$ motifs $A G, A T C, G T C, G T C, T$ and $C C, G G T, G A G, G C$ are observed.
(ii) anti-codon regions of SeC-tRNAs: The $X$ motif $S e C-t R N A X m$ $G, C T G, G A C, T T C, \underline{A A}$ is in a different frame than the anti-codon $T C A$. The $X$ motif $\underline{A A}, A T C, C A G, T T$ shifted by +1 nucleotide from $S e C-$ $t R N A X m$ is also in a different frame than the anti-codon $T C A$.
(iii) 3 'region of SeC-tRNAs: One $X$ motif $T G, A T C, T T C, C$ is observed.

### 4.2.3.17 $X$ circular code motifs in Ser-tRNAs (Table 4.22)

(i) 5' region of Ser-tRNAs: Several classes of $X$ motifs are shifted: $R, G N W, G C C, K R, \quad S e r \quad-\quad t R N A X m_{1} \quad T G, G C C, G A \underline{G, C}$, $\underline{G C}, C T G, G T A, G G$ shifted by +1 nucleotide from $\operatorname{Ser}-t R N A X m_{1}$, $G, C T G, A A C, G G$, the class of large $X$ motifs $S e r-t R N A X m_{2}$ $G, G T T, G A A, G G C, R S C, G G T, C T$ of 18 nucleotides in T. thermophilus and E. coli and K, GAA, GGC, GC factor of Ser $-t R N A X m_{2}$.
(ii) anti-codon regions of Ser-tRNAs: The very large $X$ motif $S e r-t R N A X m_{3}$ $T G, G T C, G A A, G G C, G G C, A C C, C T G, C T$ of 22 nucleotides in T. thermophilus is in a different frame than the anti-codon $G C T$. The large $X$ motif $S e r-t R N A X m_{4} A G, G G C, G C C, G G C, C T G, C T$ of 16 nucleotides in P. furiosus which is suffix of $S e r-t R N A X m_{3}$, is thus in a different frame than the anti-codon $G C T$. The class of large $X$ motifs $S e r-t R N A X m_{5}$ $G, R C C, G G T, C T C, G A A, A A C, C$ of 17 nucleotides in $E$. coli and $T$. thermophilus is in a different frame than the anti-codon $C G A$. The $X$ motif $A, C T C, G A G, A T C, C$ which is suffix of $S e r-t R N A X m_{5}$, is thus in a different frame than the anti-codon $C G A$. The class of $X$ motifs $G C, C T G, G A R, A$ and $A, C T G, G A A, A T C, G T$ which are shifted in
frame, are in a different frame than the anti-codon $G G A$. The $X$ motif Ser $-t R N A X m_{6} T T, G A A, A A C, C$ is in a different frame than the anti-codon $T G A$.
(iii) 3' region of Ser-tRNAs: The class of $X$ motifs $A A, R S Y, G K Y, R$ is shifted by +2 nucleotides from $\operatorname{Ser}-t R N A X m_{6} T T, G A A, A A C, C$. A large $X$ motif Ser - tRNAXm $\mathrm{T}_{7} G T, G T A, T A C, G G C, A A C, G T A, T$ of $18 \mathrm{nu}-$ cleotides is identified in E. coli. The $X$ motif $G, T T C, G C C, C A$ is shifted by +2 nucleotides from the class of $X$ motifs $T G, G G C, G T T, Y$. Several classes of $X$ motifs are observed: $G C, G A A, G C C$, $C A, A A, M Y C, K R Y, S, R, R R S, Y T C, S$ and $Y C, R C C, C T C, Y$.

### 4.2.3.18 $X$ circular code motifs in Thr-tRNAs (Table 4.23)

(i) 5' region of Thr-tRNAs: Several classes of $X$ motifs are shifted: CC,GGT,GKM,K, Thr - tRNAX $m_{1}$ $G, G T \underline{A, G C C}, T \underline{A}, \underline{A G, C T C, A A C, C \quad \text { shifted } \quad \text { by } \quad+2 \quad \text { nucleotides }}$ from Thr - tRNAXm $, ~ T, C A G, C A G, G T A, G A G, C A$ and its suffix $G, G T A, G A G, C$.
(ii) anti-codon regions of Thr-tRNAs: The class of $X$ motifs $T h r-t R N A X m_{2}$ $M, Y T C, G T A, A$ is in a different frame than the anti-codon $C G T$. The $X$ motif $T h r-t R N A X m_{3} T T, G T A, A T C$, shifted in frame with $T h r-$ $t R N A X m_{2}$, is in a different frame than the anti-codon $T G T$. Interestingly, the $X$ motif $G T, A A T, C A G, T A$ shifted by +1 nucleotide from $T h r-$ $t R N A X m_{3} T T, \underline{G T A, A T C}$ is, in contrast, in frame with the anti-codon $T G T$.
(iii) 3' region of Thr-tRNAs: The large $X$ motif $T h r-t R N A X m_{4}$ $A G, G G T, G A G, G T C, G C C, G G T, T$ of 18 nucleotides is identified in T. thermophilus. The class of $X$ motifs $A G, G G T, G A G, G T C, S$ is prefix of Thr - tRNAXm4. Several classes of $X$ motifs are observed: $\quad M, G W A, G G T, C, \quad C C, C A G, G T C, S$, $A G, G Y C, R Y M, R, \quad S, C A G, T T C, G A N, T, \quad Y, C T G, G G T, R K C, W$ and $K, R Y N, R Y C, G G C, W$.

### 4.2.3.19 $X$ circular code motifs in Trp-tRNAs (Table 4.24)

(i) 5' region of Trp-tRNAs: The $X$ motif $\underline{G T}, G \underline{G T}, G T \underline{A}, G C C, T$ is shifted by +1 nucleotide from $G, G G C, \underline{G T} A, \underline{G T T}, \underline{C A}$. Several classes of $X$ motifs are identified: $A G, Y T C, A A Y, T, G C, C T G, G T C, C A$, $T G, G T A, G A G, C A$ and $C C, A T C, A T C, G C$.
(ii) anti-codon regions of Trp-tRNAs: The $X$ motif $\operatorname{Tr} p-t R N A X m$ $G C, A C C, G G T, C T C, C A$ is in a different frame than the anti-codon $C C A$. The $X$ motif $G, C T C, C A G, A C C, C$ shifted in frame with $\operatorname{Tr} p-$ $t R N A X m$, is also in a different frame than the anti-codon $C C A$.
(iii) 3' region of Trp-tRNAs: The class of $X$ motifs $\underline{T G, G A G, K T Y, S \text { is shifted }}$ by +2 nucleotides from the $X$ motif $G, G G T, \overline{G T T, G G}$.

### 4.2.3.20 $X$ circular code motifs in Tyr-tRNAs (Table 4.25)

(i) 5' region of Tyr-tRNAs: The $X$ motif $G, G T A, G C C, T A$ is observed. The $X$ motif $G T, G G C, G G C, G G$ is shifted by +1 nucleotide from the $X$ motif $G C, C T G, G T A, \underline{G T}$. The $X$ motif $G, G A G, C A G, A C$ occurs before the anti-codon $G T A$.
(ii) anti-codon regions of Tyr-tRNAs: The $X$ motif Tyr - tRNAXm $m_{1}$ $G, G A C, G G T, C \underline{T G, T A}$ is in a different frame than the anti-codon $G T A$. Interestingly, the $X$ motif $\underline{T, G T A}, A A T, C T G, C$ shifted by +1 nucleotide from Tyr $-t R N A X m_{1}$ is, in contrast, in frame with the anti-codon $G T A$.
(iii) 3' region of Tyr-tRNAs: The $X$ motif $\underline{A A, A C \underline{C}, G T T, G G C, G T A, T}$ is shifted by +2 nucleotides from the $X$ motif $T, G T \underline{A}, A A T, \underline{C T G}, C$. A very large $X$ motif $T y r-t R N A X m_{2}$ $T, G C C, G T C, A T C, G A C, T T C, G A A, G G T, T$ of 23 nucleotides is identified in $E$. coli. The $X$ motif $T, G C C, G T C, A C$ is prefix of Tyr - $t R N A X m_{2}$, the $X$ motif $A, G A C, T T C, G A A, G G T, T$ is suffix of Tyr - tRNAX $m_{2}$ and the $X$ motif $A T, G C C, T T C, G C$ is factor of Tyr $-t R N A X m_{2}$.

### 4.2.3.21 $X$ circular code motifs in Val-tRNAs (Table 4.26)

(i) 5' region of Val-tRNAs: Several classes of $X$ motifs are shifted: $\quad G, G G Y, G R Y, T A, \quad T, C A G, C T G, G S, \quad$ Val $-t R N A X m_{1}$
$\underline{A G}, C \underline{T G, G T T, A T}, \quad \underline{A, G T} \underline{T, G G T, T A} \quad$ shifted $\quad$ by $+1 \quad$ nucleotide from Val - tRNAXm $, ~ A, G G T, G G C, T A, \quad$ Val $-t R N A X m_{2}$ $A, C T G, G T T, \underline{A T}, \underline{A T}, G A C, G C C, G C C, C T$ shifted by +1 nucleotide from $\mathrm{Val}-t R N A X m_{2}$ and $G C, A C C, A C C, T T$ which occurs before the anti-codon $G A C$.
(ii) anti-codon regions of Val-tRNAs: The class of large $X$ motifs Val $t R N A X m_{3} A T, G A C, G C C, R C C, C T S, A C$ of 16 nucleotides identified in $P$. furiosus is in a different frame than the anti-codons $C A C$ and $G A C$. The $X$ motif $V a l-t R N A X m_{4} A, C T C, G C C, \underline{T T}$ is in a different frame than the anti-codon $T A C$. Interestingly, the $X$ motif $T, T A C, G A G, G C$ shifted by +2 nucleotides from Val $-t R N A X m_{4}$ is, in contrast, in frame with the anti-codon TAC.
(iii) 3' region of Val-tRNAs: Several classes of $X$ motifs are observed: Val - tRNAXm $m_{5}$ A,GAG,GTC, GTA,GGT,T, R,GAG,GTY,C prefix of Val - tRNAX $m_{5}, G, G T C, G K Y, G G T, T$ suffix of Val - $t R N A X m_{5}$, $A A, G T C, C T C, T, C C, K W C, R Y C, R C C, C A$ and $T, G C C, G C C, C A$.

### 4.2.3.22 Summary of $X$ circular code motifs in tRNA SEQUENCes

We mention some of the properties of $X$ motifs found in tRNAs:
(i) an $X$ motif can occur at the same position in the same isoaccepting tRNA of different species, e.g. the $X$ motif $C A G, G A G, G T C$ is at position 40 in Ala - tRNA of E. coli and T. thermophilus (Table 4.6), the $X$ motif $T, C A G, C T G, G A T, A$ is at position 12 in $\operatorname{Arg}-t R N A$ of $E$. coli and T. thermophilus (Table 4.7), etc.
(ii) an $X$ motif can occur at the same position in different isoaccepting tRNAs, e.g. the $X$ motif $T, C A G, C T G, G$ is at position 12 in $A l a-t R N A$ of E. coli, Arg - tRNA of E. coli and T. thermophilus, His $-t R N A$ of T. thermophilus, Ile $-t R N A$ of T. thermophilus and Lys $-t R N A$ of $T$. thermophilus, Val $-t R N A$ of $E$. coli and T. thermophilus (Table 4.6,4.7,4.14,4.15,4.17 and 4.26), the $X$ motif $G T A, G T T, C A G$ is at the same position 7 in $\operatorname{Arg}-t R N A, A s n-t R N A, A s p-t R N A, G l y-t R N A$, $\operatorname{Tr} p-t R N A$ of $E$. coli (Table 4.7, 4.8, 4.9, 4.13 and 4.24), etc.
(iii) an $X$ motif can be shifted by $0,+1$ or $+2 \bmod 3$ nucleotides from another $X$ motif in the same species or in different species.
(a) an $X$ motif can be in the same frame as the anticodon, e.g. the very large $X$ motif $A l a-t R N A X m_{1}$ $G C, C T C, A A T, G G C, A T T, G A G, G A G, G T C, A$ of 24 nucleotides in Ala - $t R N A$ of $T$. thermophilus is in frame with the anti-codon $G G C$ (Table 4.6), the $X$ motif $G, A T T, C T G, A T T, C$ in $G l n-t R N A$ of $E$. coli is in frame with the anti-codon $C T G$ (Table 4.11), etc.
(b) an $X$ motif can be in a different frame than the anti-codon with a shift of one nucleotide, e.g. the $X$ motif Cys $-t R N A X m_{3}$ $A, C T G, C A G, A T C, C$ in Cys $-t R N A$ of $P$.furiosus is shifted by +1 nucleotide relative to the anti-codon $G C A$ (Table 4.10), etc.
(iv) an $X$ motif can be in a different frame than the anti-codon with a shift of two nucleotides, e.g. the large $X$ motif $\operatorname{Arg}-t R N A X m_{6}$ $A, G T A, C T C, G G C, T A C, G A A, C$ of 17 nucleotides in $A r g-t R N A$ of $E$. coli is shifted by $+2(-1)$ nucleotides relative to the anti-codon $A C G$ (Table 4.7), etc.

### 4.2.3.23 Coverage of $X$ circular code motifs in prokaryotic tRNAs

Table 4.27 shows that the coverage (Equation 3.5) of $X$ motifs (Algorithm 3.2) is greater in the 5 ' region of tRNAs in E. coli, T. thermophilus and P. furiosus (mean equal to $88 \%$, minimum equal to $62 \%$ and maximum equal to $100 \%$ ) compared to their 3 ' region (mean equal to $71 \%$, minimum equal to $23 \%$ and maximum equal to $98 \%$ ). The coverage of $X$ motifs is maximal ( $100 \%$ ) in the $5^{\prime}$ region of tRNAs of Gly, Leu, Pro and Thr, and minimal (around 30\%) in the 3' region of tRNAs of Asp, Glu, His and SeC. This indicates a possible role of $X$ circular code in the formation of tRNA sequences, or the possibility of $X$ motifs in tRNAs interacting with $X$ motifs in rRNA to help with positioning on the A, $P$ and $E$ sites of the ribosome.

### 4.3 Analysis of $X$ CIRCULAR CODE motifs in eukaryotic Genomes

The motifs studied in this section are large, having a minimum length of 30 nucleotides (Equation 3.4) which were extracted using the Frames algorithm (Algorithm 3.2) with the eukaryotic genomes (Table 3.2).

Table 4.27: The lengths and the anti-codon positions are average values in each isoaccepting tRNA. The coverage of $X$ motifs is given in rounded percentage.

|  |  | Coverage ofmotifs $(\%)$ <br> tRNA |  |  |
| :--- | ---: | ---: | ---: | ---: |
| Length | anti-codon position | 5 | region | 3 |
| region |  |  |  |  |
| Ala | 76 | 34 | 67 | 63 |
| Arg | 77 | 35 | 97 | 88 |
| Asn | 76 | 34 | 91 | 63 |
| Asp | 78 | 36 | 80 | 23 |
| Cys | 74 | 33 | 94 | 77 |
| Gln | 75 | 34 | 85 | 77 |
| Clu | 77 | 34 | 91 | 32 |
| Cly | 76 | 35 | 100 | 95 |
| His | 77 | 35 | 85 | 35 |
| Ile | 77 | 35 | 62 | 93 |
| Leu | 87 | 36 | 100 | 94 |
| Lys | 78 | 34 | 76 | 69 |
| Met | 77 | 35 | 91 | 98 |
| Phe | 76 | 34 | 85 | 70 |
| Pro | 78 | 36 | 100 | 93 |
| SeC | 95 | 35 | 88 | 31 |
| Ser | 89 | 35 | 88 | 92 |
| Thr | 77 | 34 | 100 | 95 |
| Trp | 77 | 35 | 94 | 57 |
| Tyr | 83 | 35 | 85 | 59 |
| Val | 78 | 35 | 94 | 90 |
| Mean | 79 | 35 | 88 | 71 |
| Min | 74 | 33 | 62 | 23 |
| Max | 95 | 36 | 100 | 98 |

### 4.3.1 Occurrence of large randoms code motifs in eukaryotic GENOMES

The mean number $\bar{N}(m(R))=\frac{1}{\operatorname{Card}(R)} \sum_{j=1}^{\operatorname{Card}(R)} N\left(m\left(R_{j}\right)\right)$ and its standard deviation $\sigma(m(R))$ of large $R$ motifs $m(R)$ from $\operatorname{Card}(R)=30$ random codes are determined in the 138 eukaryotic genomes. The computation leads to $\bar{N}(m(R))=1171$ and $\sigma(M(R))=1170$. By assuming a normal distribution of the population, a student $t$ test gives a confidence interval at $99 \%$ for the mean $\bar{N}(m(R))$ equal to $[582,1760]$ (shown in Figure 4.14). Note that the number of random codes was limited to 30 as their statistical analysis in the 138 eukaryotic genomes stretches into days.


Figure 4.14: Occurrence numbers $N(m(X))$ of large $X$ motifs $m(X), N(m(\Pi)))$ of its 23 large bijective motifs $m(\Pi(X)), N\left(m\left(X_{1}\right)\right)$ and $N\left(m\left(X_{2}\right)\right)$ of its two large permuted motifs $m\left(X_{1}\right)$ and $m\left(X_{2}\right)$, respectively,in the 138 eukaryotic genomes. All these 26 classes of large motifs have lengths $l \geq 15$ trinucleotides and cardinality (composition) Card $\geq 10$ trinucleotides. The top horizontal line (1760) and the bottom horizontal line (582) represent the confidence interval at $99 \%$ (student $t$ test by assuming a normal distribution of the population) of the mean occurrence number $\bar{N}(m(R))=1171$ (standard deviation $\sigma(m(R))=1170$ ) of large $R$ motifs $m(R)$ from $\operatorname{Card}(R)=30$ random codes in the 138 eukaryotic genomes. The large $X$ motifs $m(X)$ have the highest occurrence. The six large bijective motifs $m\left(\pi_{2}(X):(A, G)\right), m\left(\pi_{4}(X):(C, G)\right), m\left(\pi_{5}(X):(C, T)\right), m\left(\pi_{9}(X):(A, T)(C, G)\right)$, $m\left(\pi_{11}(X):(A, C, T)\right)$ and $m\left(\pi_{15}(X):(A, T, G)\right)$, and the two large permuted motifs $m\left(X_{1}\right)$ and $m\left(X_{2}\right)$ have occurrence numbers greater than $\bar{N}(m(R))+2.75 \sigma(m(R)) \approx 4400$.

### 4.3.2 Occurrence of large motifs from $X, X_{1}, X_{2}$ and the 23 biJective transformations of $X$ in eukaryotic genomes

Figure 4.14 shows the occurrence numbers $N(m(X))$ of large $X$ motifs $m(X)$, $N(m(\Pi(X)))$ of its 23 large bijective motifs $m(\Pi(X)), N\left(m\left(X_{1}\right)\right)$ and $N\left(m\left(X_{2}\right)\right)$ of its two large permuted motifs $m\left(X_{1}\right)$ and $m\left(X_{2}\right)$, respectively, in the 138 eukaryotic genomes. All these 26 classes of large motifs have lengths $l \geq 15$ trinucleotides and cardinality (composition) Card $\geq 10$ trinucleotides. The large $X$ motifs $m(X)$ have the highest occurrence with $N(m(X))=7133$ compared to all the 25 other classes of large motifs $m(\Pi(X)), m\left(X_{1}\right)$ and $m\left(X_{2}\right)$ in genomes of eukaryotes. Eight large motifs also occur significantly with numbers


Figure 4.15: Occurrence numbers $N(m(X))$ of large $X$ motifs $m(X), N(m(\Pi)))$ of its six large bijective motifs $m\left(\pi_{2}(X):(A, G)\right), m\left(\pi_{4}(X):(C, G)\right), m\left(\pi_{5}(X):(C, T)\right), m\left(\pi_{9}(X):(A, T)(C, G)\right)$, $m\left(\pi_{11}(X):(A, C, T)\right)$ and $m\left(\pi_{15}(X):(A, T, G)\right), N\left(m\left(X_{1}\right)\right)$ and $N\left(m\left(X_{2}\right)\right)$ of its two large permuted motifs $m\left(X_{1}\right)$ and $m\left(X_{2}\right)$ (greater than $\bar{N}(m(R))+2.75 \sigma(m(R)) \approx 4400$, see Figure 4.14) as a function of their lengths $l$ varying from 15 to 21 trinucleotides in the 138 eukaryotic genomes. All these classes of large motifs have a cardinality $C$ ard $\geq 10$ trinucleotides (Equation 3.4). The large $X$ motifs have the highest occurrence for all trinucleotide lengths.
greater than $\bar{N}(m(R))+2.75 \sigma(m(R)) \approx 4400($ where $\bar{N}(m(R))$ and $\sigma(m(R))$ are given in Section 4.3.1). They are in descending order: $m\left(\sigma_{4}(X):(A, T)(C, G)\right)$ with $N\left(m\left(\sigma_{4}(X)\right)\right)=5447, m\left(\sigma_{15}(X):(A, T, G)\right)$ with $N\left(m\left(\sigma_{15}(X)\right)\right)=5374$, $m\left(\sigma_{11}(X):(A, C, T)\right)$ with $N\left(m\left(\sigma_{11}(X)\right)\right)=5341, m\left(X_{2}\right)$ with $N\left(m\left(X_{2}\right)\right)=$ 5289, $m\left(X_{1}\right)$ with $N\left(m\left(X_{1}\right)\right)=5223, m\left(\pi_{5}(X):(C, T)\right)$ with $N\left(m\left(\pi_{5}(X)\right)=\right.$ 4466 and $m\left(\pi_{2}(X):(A, G)\right)$ with $N\left(m\left(\pi_{2}(X):(A, G)\right)\right.$ with $N\left(m\left(\pi_{2}(X)\right)\right)=$ 4404 (Figure 4.14). Note that $\pi_{2}(X), \pi_{4}(X)$ and $\pi_{5}(X)$ are symmetric bijective transformation circular codes $\Pi_{\mathcal{S}, 2}(X)$ at 2 letters, $\pi_{9}(X)$ is a symmetric bijective transformation circular code $\Pi_{\mathcal{S}, 2,2}(X)$ of two disjoint transformations at 2 letters, and $\pi_{11}(X)$ and $\pi_{15}(X)$ are asymmetric bijective transformation circular codes $\Pi_{\mathcal{A}, 3}(X)$ at 3 letters. Note also that $\pi_{4}(X)$ and $\pi_{9}(X)$ are $C^{3}$ self-complementary trinucleotide circular codes.

The six motifs $m\left(\pi_{3}(X):(A, T)\right), m\left({ }_{12}(X):(A, G, C)\right), m\left(\pi_{16}(X):\right.$


Figure 4.16: Occurrence numbers $N(m(X))$ of large $X$ motifs $m(X), N(m(\Pi)))$ of its six large bijective motifs $m\left(\pi_{2}(X):(A, G)\right), m\left(\pi_{4}(X):(C, G)\right), m\left(\pi_{5}(X):(C, T)\right), m\left(\pi_{9}(X):(A, T)(C, G)\right)$, $m\left(\pi_{11}(X):(A, C, T)\right)$ and $m\left(\pi_{15}(X):(A, T, G)\right), N\left(m\left(X_{1}\right)\right)$ and $N\left(m\left(X_{2}\right)\right)$ of its two large permuted motifs $m\left(X_{1}\right)$ and $m\left(X_{2}\right)$ (greater than $\bar{N}(m(R))+2.75 \sigma(m(R)) \approx 4400$, see Figure 4.14) as a function of their cardinality (composition) Card varying from 10 to 15 trinucleotides in the 138 eukaryotic genomes. All these classes of large motifs have a cardinality $\operatorname{Card} \geq 10$ trinucleotides (Equation 3.4). The large $X$ motifs have the highest Occurrence for all trinucleotide cardinalities.
$(C, G, T)), \quad m\left(\pi_{18}(X) \quad: \quad(A, C, G, T)\right), \quad m\left(\pi_{19}(X) \quad: \quad(A, C, T, G)\right) \quad$ and $m\left(\pi_{23}(X):(A, T, G, C)\right)$ occur randomly $\left(N\left(m\left(\pi_{i}(X)\right)\right) \in[582,1760], i=\right.$ $3,12,16,18,19,23$, see section 4.3.1) and the four motifs $m\left(\pi_{10}(X):(A, C, G)\right)$, $m\left(\pi_{13}(X):(A, G, T)\right), m\left(\pi_{14}(X):(A, T, C)\right)$ and $m\left(\pi_{17}(X):(C, T, G)\right)$ have low occurrences $\left(2000<N\left(m\left(\pi_{i}(X)\right)<2400, i=10,13,14,17\right)\right.$ (Figure 4.14).

Figures 4.15 and 4.16 strengthen the above mentioned results. Indeed, 4.15 shows that the large $X$ motifs with cardinality Card $\geq 10$ trinucleotides (Equation 3.4) have the highest occurrence compared to all the 25 other classes of large motifs $m(\Pi(X)), m\left(X_{1}\right)$ and $m\left(X_{2}\right)$ for all lengths $l$ from 15 to 21 trinucleotides. Figure 4.16 shows that the large $X$ motifs with length $l \geq 15$ trinucleotides (Equation 3.4) have the highest occurrence compared to all the 25 other classes of large motifs $m(\Pi(X)), m\left(X_{1}\right)$ and $m\left(X_{2}\right)$ (with length $l \geq 15$ trinucleotides) for all cardinalities Card from 10 to 15 trinucleotides.


Figure 4.17: Base ratio $r\left(\mathcal{G}\right.$ (Equation $3.7 \mathrm{in} \%$ ) of coding/non-coding regions and base ratio $r_{m(X)}(\mathcal{G})$ (Equation 3.9) of $X$ motifs $m(X)$ of length $l \geq 10$ trinucleotides and cardinality (composition) Card $\geq 5$ trinucleotides (Equation 3.8) in coding/non-coding regions of the 138 eukaryotic genomes $\mathcal{G}$. The vertical red line $r(\mathcal{G})=100 \%$ makes a partition of genomes $\mathcal{G}$ according to their base content in coding regions. When $r(\mathcal{G})<100 \%$, the total base number $N\left(\mathcal{G}_{C}\right)$ of coding regions $\mathcal{G}_{C}$ in genome $\mathcal{G}$ is less than the total base number $N\left(\mathcal{G}_{\bar{C}}\right)$ of all non-coding regions $\mathcal{G}_{\bar{C}}$ in $\mathcal{G}$, and conversely when $r(\mathcal{G})>100 \%$. The genome $\mathcal{G}=$ Plasmodium falciparum with $r_{m(X)}(\mathcal{G})=79.3$ is not represented in the figure. There is no correlation between $r(\mathcal{G})$ and $r_{m(X)}(\mathcal{G})$.

### 4.3.3 Largest $X$ motifs in eukaryotic genomes

Table 4.28 shows the top 20 largest $X$ motifs $m_{\mathcal{G}_{C h r}}(X)$ with cardinality (composition) Card $\geq 10$ trinucleotides (Equation 3.4) in the chromosomes $\mathcal{G}_{C h r}$ of the 138 eukaryotic genomes $\mathcal{G}$ in descending order of their length (in trinucleotides) $l \geq 45$. The $1^{\text {st }}$ largest $X$ motif $m_{\text {Solanum }_{3}}(X)$ is observed in a non-coding region of the chromosome $C h r=3$ in the genome $\mathcal{G}=$ Solanum pennellii. It has a length of $l=155$ trinucleotides (456 nucleotides) and an expectation $E\left[N\left(m_{\text {Solanum }_{3}}(X)\right)\right]=10^{-71}$ (Equation 3.6). The $2^{\text {nd }}$ and $3^{\text {rd }}$ largest $X$ motifs $m_{\text {Salmo }_{15}}(X)$ are found in non-coding regions of chromosome $C h r=15$ in the $\mathcal{G}=$ Salmosalar genome. They have a different composition but the same length $l=118$ trinucleotides (354 nucleotides) and an expectation $E\left[N\left(m_{\text {Solmo }_{1} 5}(X)\right)\right]=10^{-52}$. The biological function and evolution of these

### 4.3. ANALYSIS OF $X$ CIRCULAR CODE MOTIFS IN EUKARYOTIC

Table 4.28: The top 20 largest $X$ motifs $m_{\mathcal{G}_{C h r}}(X)$ with cardinality (compostion) $C a r d \geq 10$ trinucleotides (Equation 3.4) in the chromosomes $\mathcal{G}_{C h r}$ of the 138 eukaryotic genomes $\mathcal{G}$ in descending order of length (in trinucleotide) $l \geq 45$. The $1^{s t}$ and $2^{\text {nd }}$ columns give the genome $\mathcal{G}$ and its chromosome number $\mathcal{G}_{C h r}$, respectively, the $3^{r d}$ column gives its bases size $N\left(\mathcal{G}_{C h r}\right)$, the $4^{t h}$ and $5^{t h}$ columns indicate the start and end positions of the large $X$ motif, the $6^{t h}$ column gives the length $l$ in trinucleotide, the $7^{\text {th }}$ column indicates its expectation $E$ (Equation 3.6) and the last column shows if the motif is in a coding region (Yes) or non-coding region (No).

| Genome $\mathcal{G}$ | $\mathcal{G}_{C h r}$ | Size (in bases) $N\left(\mathcal{G}_{C h r}\right)$ | Start | End | Length <br> (in trinucleotides) | Expectation $E$ | In coding region |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Solanum pennellii | 3 | 75414019 | 36982714 | 36983178 | 155 | $10^{71}$ | No |
| Salmo salar | 15 | 103963436 | 16024777 | 16025130 | 118 | $10^{52}$ | No |
| Salmo salar | 15 | 103963436 | 17850373 | 17850726 | 118 | $10^{52}$ | No |
| Monodelphis domestica | 2 | 541556283 | 513328228 | 513328533 | 102 | $10^{43}$ | No |
| Solanum lycopersicum | 8 | 65866657 | 30359989 | 30360276 | 96 | $10^{41}$ | No |
| Monodelphis domestica | 4 | 435153693 | 290107123 | 290107407 | 95 | $10^{40}$ | No |
| Plasmodium falciparum | 11 | 2038337 | 872956 | 873216 | 87 | $10^{38}$ | Yes |
| Equus caballus | 28 | 46177339 | 35484817 | 35485047 | 77 | $10^{32}$ | No |
| Bombus terrestris | 14 | 11649563 | 11165956 | 11166153 | 66 | $10^{27}$ | Yes |
| Sorghum bicolor | 4 | 68034345 | 38474677 | 38474856 | 60 | $10^{23}$ | No |
| Felis catus | 3 | 140925898 | 2211844 | 2212020 | 59 | $10^{22}$ | No |
| Cynoglossus semilaevis | 9 | 19616557 | 14919031 | 14919192 | 54 | $10^{20}$ | No |
| Plasmodium knowlesi | 13 | 2200295 | 1265167 | 1265322 | 52 | $10^{20}$ | Yes |
| Mus musculus | 1 | 195471971 | 74368813 | 74368968 | 52 | $10^{18}$ | Yes |
| Micromonas sp. | 12 | 1084119 | 530353 | 530496 | 48 | $10^{19}$ | Yes |
| Dictyostelium discoideum | 2 | 8484197 | 1796161 | 1796304 | 48 | $10^{18}$ | Yes |
| Apis mellifera | 4 | 12718334 | 12440101 | 12440241 | 47 | $10^{17}$ | No |
| Salmo salar | 19 | 82978132 | 46877047 | 46877184 | 46 | $10^{16}$ | No |
| Bombus terrestris | 15 | 11467329 | 3286219 | 3286353 | 45 | $10^{16}$ | No |
| Camelina sativa | 10 | 25316904 | 13177546 | 13177680 | 45 | $10^{16}$ | No |

unexpected large $X$ motifs in the eukaryotic genomes are unknown.

Table 4.29: Largest $X$ motifs $m_{\mathcal{H} C h r}(X)$ with cardinality (compostion) $C a r d \geq 10$ trinucleotides (Equation 3.4) and expectation $E<1$ (Equation 3.6) in the chromosomes $\mathcal{H}_{C h r}$ of the human genome $\mathcal{G}=\mathcal{H}=$ Homo sapiens. The $1^{\text {st }}$ and $2^{\text {nd }}$ columns give the human chromosome number $\mathcal{H}_{C h r}$ and its base size $N\left(\mathcal{H}_{C h r}\right)$, respectively, the $3^{r d}$ column shows the largest $X$ motif with cardinality Card $\geq 10$ trinucleotides and expectation $E<1$, the $4^{\text {th }}$ and $5^{\text {th }}$ columns indicate the start and end positions of the large $X$ motif, the $6^{\text {th }}$ column gives the length $l$ in trinucleotide, the $7^{\text {th }}$ column indicates its expectation $E$ (Equation 3.6 ) and the last column shows if the motif is in a coding region (Yes) or non-coding region (No).

| $\mathcal{H}_{C h r}$ | size (in bases) $N\left(\mathcal{H}_{C h r}\right)$ | $X$ motifs | Start | End | Length (in trinucleotides) | Expectation $E$ | In coding region |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 248956422 | GAG,GAG,GAG,CTG,CTG,GCC,CAG,CTG,GAG,GAG,TAC,GAG,CAG,GTC,ATC,CTG,GAC,TTC, CAG,TTC,AAC,CTG,GAG,GCC,ACC | 3763375 | 3763449 | 25 | $5.9 \times 10^{-5}$ | Yes |
| 2 | 242193529 | GTC,GAT,GAG,CAG,AAT,GCC,CAG,ACC,CAG,GAG,CAG,GAG,GGC,TTC,GTC,CTG,GGC,CTC | 233449984 | 233450037 | 18 | $2.0 \times 10^{-1}$ | Yes |
| 4 | 190214555 | GCC,ATC,ATT,ATC,ATT,ATC,ATC,CTC,ACC,TTC,ATC,ATT,AAT,AAC,CTG,GGC,CAG,GGT | 42018853 | 42018906 | 18 | $1.5 \times 10^{-1}$ | No |
| 5 | 181538259 | GAA,ATC,TTC,ATC,ATT,ACC,CTC,ACC,GCC,GCC,ATC,ATT, GAC, CTG, GTT,AAT,GTT | 133306903 | 133306953 | 17 | $4.7 \times 10^{-1}$ | No |
| 7 | 159345973 | ATC,ACC,CAG,GAT,GAA,GAT,GGT,CTC,ACC,CTG,CTC,ATT,GAG,GAT,GCC,GGT,GGT | 30452806 | 30452856 | 17 | $4.1 \times 10^{-1}$ | Yes |
| 8 | 145138636 | ACC,GTC,ACC,AAC,CTG,TTC,ATC,CTC,AAC,CTG,GCC,ATC,GCC,GAC,GAG,CTC,TTC | 52940113 | 52940163 | 17 | $3.8 \times 10^{-1}$ | Yes |
| 9 | 138394717 | GGT, CTC, CAG,GCC,AAT,GTC,ATT,GAC,GTC,ACC,ATC,ATC,GCC,ATC,ACC,ATC,ATT,ACC | 95705686 | 95705739 | 18 | $1.1 \times 10^{-1}$ | No |
| 11 | 135086622 | GAT,GAT,GCC,ACC,ACC,CTC,TAC,CTG,CAG,AAC,AAC,CAG,ATC,AAC,AAC,GCC,GGC,ATC | 64116508 | 64116561 | 18 | $1.1 \times 10^{-1}$ | Yes |
| 13 | 114364328 | AAT,GAG,GAC,ACC,ACC,CAG,GGC,ATC,GCC,AAC,GAG,GAA,GCC,GCC,CAG,GGC,ATC,GCC, GAG,GAC,GCC,ATC,CAG,GGC,ATC,GCC,AAC,GAG,GAG,GTT,GCC,CAG,GGC,ATC,GCC,AAT | 18235684 | 18235791 | 36 | $7.5 \times 10^{-11}$ | No |
| 14 | 107043718 | GCC,CAG,GAC,GAC,GAG,GGT,CTG,CTG,GAC,AAC,TTC,GTC,ACC,TTC,TTC,ATT | 99716146 | 99716193 | 16 | $8.9 \times 10^{-1}$ | Yes |
| 15 | 101991189 | GGC,GAA,GAA,GGT,GAA,GAT,GAA,GAG,GAT,GAA,GAT,CTG,GCC,CTC,GGT, GAC,CAG,GTA | 68208355 | 68208408 | 18 | $8.2 \times 10^{-2}$ | Yes |
| 17 | 83257441 | CTG,CTG,GTT,GAA,GTT,GTC,AAT,GAT,GAC,GCC,AAT,GAA,GAG,GTT,GAG,GGT,GAA,GAA | 63944680 | 63944733 | 18 | $6.7 \times 10^{-2}$ | Yes |
| 18 | 80373285 | ATC,GAG,CAG,AAT,GCC,ACC,AAC,ACC,TTC,CTG,GTC,TAC,ACC,GAG,GAG,GAC | 49583566 | 49583613 | 16 | $6.6 \times 10^{-1}$ | Yes |
| 19 | 58617616 | GAA,ACC,AAC,CAG,GTC,CTC,ATC,AAC,ATT,GGC,CTG,CTG,CTC,CTG,GCC,TTC | 13959991 | 13960038 | 16 | $4.8 \times 10^{-1}$ | Yes |
| 20 | 64444167 | TAC,CTG,GCC,CAG,GTC,CAG,GGT,GAC,GTT,GAC,CTC,GTT,GTA,CTC,CAG,GCC | 62362396 | 62362443 | 16 | $5.3 \times 10^{-1}$ | No |
| 22 | 50818468 | CAG,GTT,GAA,GAA,GTT,GTA,GTT,GCC,GGT,GAT,GAT,AAT, CAG,GAC,CTG,CAG,CAG | 50505760 | 50505810 | 17 | $1.3 \times 10^{-1}$ | Yes |
| X | 156040895 | CTC,CAG,GTA,GAG,GGC,ATT,GAG,CAG,CTC,AAT,GAT,GTC,AAC,GAG,GAC,CTG,GTT,GTC | 39981361 | 39981414 | 18 | $1.3 \times 10^{-1}$ | No |



Figure 4.18: Base ratio $r\left(\mathcal{G}_{C h r}\right)$ (Equation 3.7 in \%) of coding/non-coding regions and base ratio $r_{m(X)}\left(\mathcal{H}_{C h r}\right)$ (Equation 3.9) of $X$ motifs of length $l \geq 10$ trinucleotides and cardinality (composition) Card $\geq 5$ in coding/non-coding regions of the 23 chromosomes $\mathcal{H}_{C h r}$ in the human genome $\mathcal{G}=\mathcal{H}=$ Homo sapiens. There is no correlation between $r\left(\mathcal{G}_{C h r}\right)$ and $r_{m(X)}\left(\mathcal{H}_{C h r}\right)$.

### 4.3.4 Largest $X$ motifs in Homo sapiens

Table 4.29 shows the largest $X$ motifs $m_{\mathcal{H}_{C h r}}(X)$ with cardinality (composition) Card $\geq 10$ trinucleotides (Equation 3.4) and expectation $E<1$ (Equation 3.6) in the chromosomes $\mathcal{H}_{C h r}$ of the human genome $\mathcal{G}=\mathcal{H}=$ Homo sapiens. The largest $X$ motif $m_{\mathcal{H}_{13}}(X)$ is found in a non-coding region of the human chromosome $C h r=13$. it has a length of $l=36$ trinucleotides and an expectation $E\left[N\left(m_{\text {Solmo }_{1} 5}(X)\right)\right]=7.5 \times 10^{-11}$ (Equation 3.6).

### 4.3.5 $X$ motifs in Coding regions versus non-Coding regions in euKARYOTIC GENOMES

The maximal $C^{3}$ self-complementary trinucleotide circular code $X$ is a wellknown coding property of genes. Indeed, it is observed in genes of bacteria, eukaryotic, plasmids and viruses (Arquès and Michel, 1996; Michel, 2015).

Table 4.30 gives the base ratio $r(\mathcal{G})$ (Equation 3.7 in \%) of coding/non-coding

Table 4.30: Base ratio $r(\mathcal{G})$ (Equation 3.7 in \%) of coding/non-coding regions and base ratio $r_{m(X)}(\mathcal{G})$ (Equation 3.9) of $X$ motifs $m(X)$ of length $l \geq 10$ trinucleotides and cardinality (composition) Card $\geq 5$ trinucleotides (Equation 3.8) in the 138 eukaryotic genomes $\mathcal{G}$.

| Genome $\mathcal{G}$ | $r(\mathcal{G )}(\%)$ | $r_{m(X)}(\mathcal{G}) \mid$ | $\mid$ Genome $\mathcal{G}$ | $r(\mathcal{G})(\%)$ | $r_{m(X)}(\mathcal{G}) \mid$ | Genome $\mathcal{G}$ | $r(\mathcal{G )}(\%)$ | $r_{m(X)}(\mathcal{G})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anolis carolinensis | 1.6 | 5.3 | Esox lucius | 5.5 | 12.2 | Ovis aries | 1.3 | 20.0 |
| Anopheles gambiae | 8.6 | 15.6 | Felis catus | 1.4 | 19.4 | Pan paniscus | 1.1 | 9.4 |
| Apis mellifera | 8.2 | 3.5 | Ficedula albicollis | 2.5 | 19.6 | Pan troglodytes | 1.1 | 8.8 |
| Arabidopsisthaliana | 38.6 | 5.4 | Fragaria vesca | 18.5 | 5.1 | Papio anubis | 1.3 | 7.5 |
| Aspergillus fumigatus | 93.7 | 8.7 | Gallus gallus | 2.9 | 16.6 | Phaeodactylum tricornutum | 114.7 | 3.0 |
| Babesia bigemina | 196.0 | 5.2 | Glycine max | 6.9 | 4.5 | Phaseolus vulgaris | 7.2 | 4.1 |
| Babesia bovis | 213.3 | 9.1 | Gorilla gorilla | 1.2 | 9.4 | Plasmodium cynomolgi | 69.9 | 5.6 |
| Babesia microti | 263.9 | 5.4 | Gossypium raimondii | 6.3 | 6.0 | Plasmodium falciparum | 111.1 | 79.3 |
| Beta vulgaris | 7.0 | 4.0 | Homo sapiens | 1.2 | 8.4 | Plasmodium knowlesi | 90.1 | 15.6 |
| Bombus terrestris | 8.1 | 3.8 | Kazachstania africana | 239.2 | 8.4 | Plasmodium vivax | 93.1 | 16.6 |
| Bos taurus | 1.3 | 21.9 | Kluyveromyces lactis | 223.9 | 9.8 | Poecilia reticulata | 5.9 | 16.5 |
| Brachypo diumdistachyon | 14.0 | 6.6 | Komagataella phaffii | 358.7 | 6.4 | Pongo abelii | 1.1 | 9.0 |
| Brassica napus | 13.9 | 4.8 | Lachancea thermotolerans | 260.5 | 16.4 | Populus trichocarpa | 13.2 | 2.8 |
| Brassica oleracea | 12.9 | 5.0 | Leishmania braziliensis | 94.8 | 9.3 | Prunus mume | 17.3 | 5.6 |
| Brassica rapa | 23.1 | 6.4 | Leishmania donovani | 82.2 | 6.8 | Rattus norvegicus | 1.4 | 10.2 |
| Caenorhabditis briggsae | 28.9 | 6.5 | Leishmania infantum | 95.0 | 8.8 | Saccharomyces cerevisiae | 257.2 | 15.2 |
| Caenorhabditis elegans | 36.1 | 6.4 | Leishmania major | 91.6 | 8.2 | Salmo salar | 3.3 | 11.7 |
| Callithrix jacchus | 1.2 | 8.8 | Leishmania mexicana | 96.5 | 7.8 | Scheffersomyces stipitis | 125.3 | 3.3 |
| Camelina sativa | 19.7 | 4.3 | Leishmania panamensis | 90.1 | 8.1 | Schizosaccharomyces pombe | 131.4 | 6.0 |
| Candida dubliniensis | 156.4 | 3.4 | Lepisosteus oculatus | 3.7 | 21.9 | Sesamum indicum | 15.1 | 5.0 |
| Candida glabrata | 179.8 | 9.3 | Macaca fascicularis | 1.2 | 7.6 | Setaria italica | 9.8 | 7.4 |
| Candida orthopsilosis | 202.1 | 6.9 | Macaca mulatta | 1.2 | 7.6 | Solanum lycopersicum | 4.4 | 4.1 |
| Canis lupus | 1.5 | 13.0 | Magnaporthe oryzae | 70.0 | 11.8 | Solanum pennellii | 3.9 | 3.3 |
| Capra hircus | 1.2 | 20.7 | Malus domestica | 7.4 | 7.1 | Sorghum bicolor | 6.0 | 6.7 |
| Chlorocebus sabaeus | 1.3 | 7.3 | Medicago truncatula | 14.2 | 4.6 | Sus scrofa | 1.2 | 26.4 |
| Chrysemys picta | 1.3 | 8.8 | Meleagris gallopavo | 2.7 | 14.6 | Taeniopygia guttata | 2.4 | 18.9 |
| Cicer arietinum | 9.0 | 4.5 | Micromonas sp. | 228.4 | 2.4 | Takifugu rubripes | 11.3 | 10.4 |
| Ciona intestinalis | 24.8 | 6.6 | Microtus ochrogaster | 1.5 | 15.8 | Tetrapisispora blattae | 165.4 | 7.7 |
| Citrus sinensis | 13.0 | 3.6 | Monodelphis domestica | 1.0 | 4.5 | Tetrapisispora phaffii | 197.6 | 9.5 |
| Cryptococcus gattii | 124.6 | 5.4 | Mus musculus | 1.3 | 12.5 | Thalassiosira pseudonana | 119.0 | 1.5 |
| Cryptococcus neoformans | 115.2 | 6.6 | Myceliophthora thermophila | 57.4 | 18.5 | Theileria annulata | 266.6 | 8.0 |
| Cryptosporidium parvum | 298.9 | 4.6 | Nasonia vitripennis | 13.6 | 11.4 | Theileria equi | 223.3 | 3.1 |
| Cucumis sativus | 15.2 | 6.5 | Naumovozyma castellii | 286.4 | 8.6 | Theileria orientalis | 216.1 | 2.7 |
| Cyanidioschyzon merolae | 81.5 | 3.7 | Naumovozyma dairenensis | 175.6 | 6.9 | Theileria parva | 215.3 | 5.1 |
| Cynoglossus semilaevis | 9.0 | 7.2 | Neospora caninum | 44.8 | 11.8 | Theobroma cacao | 11.6 | 6.7 |
| Danio rerio | 3.4 | 7.0 | Neurospora crassa | 58.1 | 11.5 | Thielavia terrestris | 58.4 | 14.6 |
| Debaryomyces hansenii | 288.2 | 7.7 | Nomascus leucogenys | 1.2 | 8.5 | Torulaspora delbrueckii | 367.6 | 6.6 |
| Dictyostelium discoideum | 161.8 | 13.3 | Ogataea parapolymorpha | 545.6 | 7.6 | Tribolium castaneum | 11.4 | 1.2 |
| Drosophila melanogaster | 17.4 | 11.1 | Oreochromis niloticus | 5.7 | 14.8 | Trypanosoma brucei | 150.1 | 2.1 |
| Drosophila pseudoobscura | 23.9 | 7.6 | Ornithorhynchus anatinus | 1.1 | 3.2 | Ustilago maydis | 156.3 | 8.6 |
| Drosophila simulans | 14.7 | 13.6 | Oryctolagus cuniculus | 1.1 | 19.2 | Vigna radiata | 9.8 | 4.9 |
| Drosophila yakuba | 20.3 | 10.5 | Oryza brachyantha | 12.8 | 12.4 | Vitis vinifera | 8.0 | 6.0 |
| Elaeis guineensis | 4.3 | 11.2 | Oryza sativa | 8.7 | 8.1 | Yarrowia lipolytica | 85.2 | 14.9 |
| Equus caballus | 1.4 | 18.1 | Oryzias latipes | 4.9 | 18.2 | Zea mays | 2.2 | 4.0 |
| Eremothecium cymbalariae | 202.6 | 3.1 | Ostreococcus lucimarinus | 231.1 | 1.3 | Zygosaccharomyces rouxii | 319.5 | 3.6 |
| Eremothecium gossypii | 335.6 | 15.8 | Ostreococcus tauri | 437.3 | 1.6 | Zymoseptoria tritici | 56.8 | 4.0 |
|  |  |  |  |  |  | Mean | 79.2 | 9.3 |
|  |  |  |  |  |  | Median | 15.2 | 7.6 |

regions and base ratio $r_{m(X)}(\mathcal{G})$ (Equation 3.9) of $X$ motifs $m(X)$ of length $l \geq 10$ trinucleotides and cardinality (composition) Card $\geq 5$ trinucleotides (Equation 3.8) in coding/non-coding regions of the 138 eukaryotic genomes $\mathcal{G}$.

The lowest value $r(\mathcal{G})$ of coding/non-coding regions is observed in the genome of $\mathcal{G}=$ Monodelphis domestica with $r(\mathcal{G})=1.0 \%\left(r_{m(X)}(\mathcal{G})=4.5\right)$. The highest value is found in the genome of $\mathcal{G}=$ Ogataea parapolymorpha with $r(\mathcal{G})=545.6 \%$ $\left(r_{m(X)}(\mathcal{G})=7.6\right)$. The mean value is $\bar{r}(\mathcal{G})=79.2 \%$ and the median value $\widetilde{r}(\mathcal{G})=$ $15.2 \%$.

The lowest value $r_{m(X)}(\mathcal{G})$ of coding/non-coding regions is observed in the genome of $\mathcal{G}=$ Tribolium castaneum with $r_{m(X)}(\mathcal{G})=1.2(r(\mathcal{G})=11.4 \%)$. The highest value is found in the genome of $\mathcal{G}=$ Plasmodium falciparum with $r_{m(X)}(\mathcal{G})=79.3(r(\mathcal{G})=111.1 \%)$. The mean value is $\bar{r}(\mathcal{G})=9.3$ and the median value $\widetilde{r}(\mathcal{G})=7.6$.

Figure 4.17 gives a graphical representation of Table 4.30. No correlation was found between $r(\mathcal{G})$ and $r_{m(X)}(\mathcal{G})$, with sample correlation coefficient of -0.12 .

Thus, as expected according to previous works, the $X$ motifs occur preferentially in genes of genomes with a factor of about $8\left(\widetilde{r}_{m(X)}(\mathcal{G})=7.6<8<\right.$ $\left.\bar{r}_{m(X)}(\mathcal{G})=9.3\right)$. Furthermore, this circular code property is verified whatever the base content of coding regions in the genomes, with sample correlation coefficient of $r=-0.12$.

Table 4.31: Base ratio $r\left(\mathcal{H}_{C h r}\right)$ (Equation 3.7 in \%) of coding/non-coding regions and base ratio $r_{m(X)}(\mathcal{G})$ (Equation 3.9) of $X$ motifs $m(X)$ of length $l \geq 10$ trinucleotides and cardinality (composition) Card $\geq 5$ trinucleotides (Equation 3.8) in the 24 chromosomes $\mathcal{H}_{C h r}$ of the human genome $\mathcal{G}=\mathcal{H}=$ Homo sapiens.

| $\mathcal{H}_{C h r}$ | $r\left(\mathcal{H}_{C h r}\right)(\%)$ | $r_{m(X)}(\mathcal{G})$ | $\mathcal{H}_{C h r}$ | $r\left(\mathcal{H}_{C h r}\right)(\%)$ | $r_{m(X)}(\mathcal{G})$ | $\mathcal{H}_{C h r}$ | $r\left(\mathcal{H}_{C h r}\right)(\%)$ | $r_{m(X)}(\mathcal{G})$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 1.5 | 8.6 | 9 | 1.1 | 7.9 | 17 | 2.5 | 7.2 |
| 2 | 1.1 | 8.0 | 10 | 1.1 | 5.8 | 18 | 0.7 | 7.1 |
| 3 | 1.0 | 8.7 | 11 | 1.6 | 8.2 | 19 | 4.1 | 6.5 |
| 4 | 0.8 | 5.2 | 12 | 1.4 | 6.5 | 20 | 1.3 | 9.2 |
| 5 | 0.9 | 7.8 | 13 | 0.6 | 7.7 | 21 | 0.8 | 10.4 |
| 6 | 1.1 | 8.3 | 14 | 1.1 | 9.1 | 22 | 1.6 | 12.0 |
| 7 | 1.1 | 6.9 | 15 | 1.2 | 7.4 | $X$ | 0.9 | 9.7 |
| 8 | 0.8 | 7.3 | 16 | 1.7 | 6.6 | $Y$ | 0.2 | 11.9 |
|  |  |  |  |  | Mean | 1.3 | 8.1 |  |
|  |  |  |  |  | Median | 1.1 | 7.8 |  |

### 4.3.6 $X$ motifs in coding regions versus non-Coding regions in Homo sapiens

Table 4.31 gives the base ratio $r\left(\mathcal{H}_{C h r}\right)$ (Equation 3.7 in \%) of coding/non-coding region and the base ratio $r_{m(X)}\left(\mathcal{H}_{C h r}\right)$ (Equation 3.9) of $X$ motifs of length $l \geq 10$
trinucleotides and cardinality (composition) Card $\geq 5$ trinucleotides (Equation 3.8) in coding/non-coding regions of the 24 chromosomes $\mathcal{H}_{C h r}$ in the human genome $\mathcal{G}=\mathcal{H}=$ Homo sapiens.

The lowest value $r\left(\mathcal{H}_{C h r}\right)$ of coding/non-coding regions is observed in chromosome $C h r=Y$ with $r\left(\mathcal{H}_{Y}\right)=0.2 \%\left(r_{m(X)}\left(\mathcal{H}_{Y}\right)=11.9\right)$. While the highest value is found in chromosome $C h r=19$ with $r\left(\mathcal{H}_{19}\right)=4.1 \%\left(r_{m(X)}\left(\mathcal{H}_{19}\right)=6.5\right)$. The mean value is $\bar{r}\left(\mathcal{H}_{C h r}\right)=1.3 \%$ and the median value $\widetilde{r}\left(\mathcal{H}_{C h r}\right)=1.1 \%$.

Remark 4.1. These two values $\bar{r}\left(\mathcal{H}_{C h r}\right)=1.3 \%$ and $\widetilde{r}\left(\mathcal{H}_{C h r}\right)=1.1 \%$. are very close to $r(\mathcal{H})=1.2 \%$ (Table 4.30).

The lowest value $r_{m(X)}\left(\mathcal{H}_{C h r}\right)$ of $X$ motifs in coding/non-coding regions is observed in chromosome $C h r=4$ with $r_{m(X)}\left(\mathcal{H}_{4}\right)=5.2\left(r\left(\mathcal{H}_{4}\right)=0.8 \%\right)$. While the highest value is found in chromosome $C h r=22$ with $r_{m(X)}\left(\mathcal{H}_{22}\right)=12.0$ $\left(r\left(\mathcal{H}_{22}\right)=1.6 \%\right)$. The mean value is $r_{m(X)}\left(\mathcal{H}_{C h r}\right)=8.1$ and the median value $\bar{r}_{m(X)}\left(\mathcal{H}_{C h r}\right)=7.8$.

Remark 4.2. These two values $\bar{r}_{m(X)}\left(\mathcal{H}_{C h r}\right)=8.1$ and $\widetilde{r}_{m(X)}\left(\mathcal{H}_{C h r}\right)=7.8$. are very close to $r_{m(X)}(\mathcal{H})=8.4$ (Table 4.30).

Table 4.18 gives a graphical representation of Table 4.31. There is no correlation found between $r\left(\mathcal{H}_{C h r}\right)$ and $r_{m(X)}\left(\mathcal{H}_{C h r}\right)$, with sample correlation coefficient of $r=-0.26$.

As in the general case, the $X$ motifs occur preferentially in coding regions of human chromosomes with a factor of about $8\left(\widetilde{r}_{m(X)}\left(\mathcal{H}_{C h r}\right)=7.8<8<\right.$ $\left.\bar{r}_{m(X)}\left(\mathcal{H}_{C h r}\right)=8.1\right)$. Furthermore, this circular code property is also verified whatever the base content of genes in human chromosomes, with sample correlation coefficient of $r=-0.26$.

### 4.4 Analysis of unitary circular code motifs in eukaryotic GENOMES

As was shown in sections 2.3.2, 2.3.2 and 2.3.2, the unitary circular code is closely related to simple repeats. The following results were generated using the algorithm described in section 3.5.4.1 with the eukaryotic genomes data (Table 3.2)

### 4.4.1 OCCURRENCE OF REPEATED DINUCLEOTIDES IN EUKARYOTIC GENOMES

The repeated dinucleotides (Section 2.3.2) are generated from the unitary circular codes of dinucleotides (Section 3.5.1). Figures 4.19 and 4.20 give the occurrence number $N\left(d i^{+}\right)$(Equation 3.13) and the base number $B\left(d i^{+}\right)$(Equation 3.14) of all the repeated dinucleotides $d i^{n}$ of length $l=2 n \geq 30$ nucleotides $(n \geq 15)$ in the genomes of eukaryotes. The results in the two Figures 4.19 and 4.20 are altogether consistent. The repeats $(A C)^{+}$and $(G T)^{+}=(\mathcal{C}(A C))^{+}$ have the highest occurrences in the eukaryotic genomes. Then, the repeat $(A T)^{+}$(note that $\left.\mathcal{C}(A T)=A T\right)$ has a lower occurrence. The repeats $(A G)^{+}$ and $(C T)^{+}=(\mathcal{C}(A G))^{+}$have occurrences lower than $(A T)^{+}$. The repeat $(C G)^{+}$(note that $\left.\mathcal{C}(C G)=C G\right)$ is almost absent. A repeated dinucleotide $d i^{+}$and its complementary repeated dinucleotide $(\mathcal{C}(d i))^{+}$have the same occurrences in the eukaryotic genomes: $N\left((A C)^{+}\right) \approx N\left((G T)^{+}\right) \approx 659400$, $B\left((A C)^{+}\right) \approx B\left((G T)^{+}\right) \approx 28800 \mathrm{~kb}, N\left((A G)^{+}\right) \approx N\left((C T)^{+}\right) \approx 299900$ and $B\left((A G)^{+}\right) \approx B\left((C T)^{+}\right) \approx 13500 \mathrm{~kb}$ (Figures 4.19, 4.20). This property is related to the complementary property of the DNA double helix.

### 4.4.2 OCCURRENCE OF REPEATED TRINUCLEOTIDES IN EUKARYOTIC GENOMES

The repeated trinucleotides (Section 2.3.2) are generated from the unitary circular codes of trinucleotides (Section 3.5.2). Figures 4.21 and 4.22 give the occurrence number $N\left(t r i^{+}\right)$(Equation 3.13) and the base number $B\left(t r i^{+}\right)$(Equation 3.14) of all the repeated trinucleotides $t r i^{n}$ of length $l=3 n \geq 30$ nucleotides $(n \geq 10)$ in the genomes of eukaryotes. Again, the results in the two Figures 4.21 and 4.22 are altogether consistent. The repeats $(A A T)^{+}$and $(A T T)^{+}=$ $(\mathcal{C}(A A T))^{+}$have the highest occurrences in the eukaryotic genomes. Then, the following repeats are observed by decreasing order of occurrence: $(A A G)^{+}$and $(C T T)^{+}=(C(A A G))^{+},(A A C)^{+}$and $(G T T)^{+}=(C(A A C))^{+},(A T C)^{+}$and $(A T G)^{+}=\left(\mathcal{C}\left(\mathcal{P}^{2}(A T C)\right)\right)^{+}$(i.e. $(A T G)^{+}$and $(G A T)^{+}=(\mathcal{C}(A T C))^{+}$belong the same equivalence class by the circular permutation map $\mathcal{P}),(A G G)^{+}$and $(C C T)^{+}=(\mathcal{C}(A G G))^{+},(A G C)^{+}$and $(C T G)^{+}=\left(\mathcal{C}\left(\mathcal{P}^{2}(A G C)\right)\right)^{+}$(i.e. $(C T G)^{+}$ and $(G C T)^{+}=(\mathcal{C}(A G C))^{+}$belong the same equivalence class $),(A C T)^{+}$and $(A G T)^{+}=(\mathcal{C}(A C T))^{+}$, and $(A C C)^{+}$and $(G G T)^{+}=(\mathcal{C}(A C C))^{+}$. The repeats $(A C G)^{+}$and $(C G T)^{+}=(\mathcal{C}(A C G))^{+}$, and $(C C G)^{+}$and $(C G G)^{+}=(\mathcal{C}(C C G))^{+}$


Figure 4.19: Occurrence number $N\left(d i^{+}\right)$ (Equation 3.13) (descending order) of all the repeated dinucleotides $d i^{n}$ of length $l=2 n \geq 30$ nucleotides ( $n \geq 15$ ) in the eukaryotic genomes. The repeated dinucleotides are generated from the unitary circular codes of dinucleotides.


Figure 4.20: Base number $B\left(d i^{+}\right)$(Equation 3.14) (descending order) of all the repeated dinucleotides $d i^{n}$ of length $l=2 n \geq 30$ nucleotides ( $n \geq 15$ ) in the eukaryotic genomes. The repeated dinucleotides are generated from the unitary circular codes of dinucleotides.
are almost absent.
A repeated trinucleotide $t r i^{+}$and its complementary repeated trinucleotide $(\mathcal{C}(\text { tri }))^{+}$have the same occurrences in the eukaryotic genomes: $N\left((A A T)^{+}\right) \approx$ $N\left((A T T)^{+}\right) \approx 95700, B\left((A A T)^{+}\right) \approx B\left((A T T)^{+}\right) \approx 4500 \mathrm{~kb}, N\left((A A G)^{+}\right) \approx$ $N\left((C T T)^{+}\right) \approx 24500$ and $B\left((A A G)^{+}\right) \approx B\left((C T T)^{+}\right) \approx 1600 \mathrm{~kb}$, etc. (Figures 4.21 and 4.22$)$. This property is again related to the complementary property of the DNA double helix. This result is also confirmed by the correlation matrix of the base number $B\left(t r i^{+}\right)$of all the repeated trinucleotides $t r i^{+}$(Table 4.33). The highest correlation is always observed between the repeated trinucleotides tri ${ }^{+}$and $(\mathcal{C}(\text { tri }))^{+}$in the eukaryotic genomes. There is no significant correlation between a repeated trinucleotide $t r i^{+}$and the size of genomes as well as the count of $A, C, G$ and $T$ and $G C$ content of genomes.

A second property is identified with the repeated trinucleotides tri ${ }^{+}$and $(\mathcal{C}(\text { tri }))^{+}$. Indeed, the repeated trinucleotides tri ${ }^{+}$and $(\mathcal{C}(\text { tri }))^{+}$have increasing occurrences in the eukaryotic genomes conversely to their number of hydrogen


Figure 4.21: Occurrence number $N\left(t r i^{+}\right)$(Equation 3.13) (descending order) of all the repeated trinucleotides $\operatorname{tr} i^{n}$ of length $l=3 n \geq 30$ nucleotides ( $n \geq 10$ ) in the eukaryotic genomes. The repeated trinucleotides are generated from the unitary circular codes of trinucleotides.
bonds (two hydrogen bonds between A and $\mathrm{T}=\mathcal{C}(\mathrm{A})$ and three hydrogen bonds between C and $\mathrm{G}=\mathcal{C}(\mathrm{C})$ ), from the highest occurrences for the two repeats $(A A T)^{+}$and $(A T T)^{+}$with a total of six hydrogen bonds to the lowest occurrences for the two repeats $(C C G)^{+}$and $(C G G)^{+}$with a total of nine hydrogen bonds.

### 4.4.3 OCCURRENCE OF REPEATED TETRANUCLEOTIDES IN EUKARYOTIC GENOMES

The repeated tetranucleotides (Section 3.5.3) are generated from the unitary circular codes of tetranucleotides (Section 2.3.2). Figures 4.23 and 4.24 give the occurrence number $N\left(\right.$ tetra $\left.^{+}\right)$(Equation 3.13) and the base number $B\left(\right.$ tetra $\left.^{+}\right)$ (Equation 3.14) of the repeated tetranucleotides tetra $^{n}$ of length $l=4 n \geq 28$ nucleotides $(n \geq 7)$ in the eukaryotic genomes. The results in the two Figures 4.23 and 4.24 are consistent and identify two classes of repeated tetranucleotides with higher occurrences. The 1st class with the highest occurrences in the eukaryotic genomes contains eight repeated tetranucleotides, by descending


Figure 4.22: Base number $N\left(t r i{ }^{+}\right)$(Equation 3.14) (descending order) of all the repeated trinucleotides $t r i^{n}$ of length $l=3 n \geq 30$ nucleotides ( $n \geq 10$ ) in the eukaryotic genomes. The repeated trinucleotides are generated from the unitary circular codes of trinucleotides.
order: $(A A A T)^{+}$and $(A T T T)^{+}=(\mathcal{C}(A A A T))^{+},(A A A G)^{+}$and $(C T T T)^{+}=$ $(\mathcal{C}(A A A G))^{+},(A G A T)^{+}$and $(A T C T)^{+}=(\mathcal{C}(A G A T))^{+}$, and $(A A G G)^{+}$and $(C C T T)^{+}=(\mathcal{C}(A A A G))^{+}$(Figure 4.23). Note that this repeated tetranucleotide order is different in Figure 4.24. The 2nd class with higher occurrences in the eukaryotic genomes contains 12 repeated tetranucleotides, by descending order: $(A T C C)^{+}$and $(A T G G)^{+}=\left(\mathcal{C}\left(\mathcal{P}^{2}(A T C C)\right)\right),(A A A C)^{+}$and $(G T T T)^{+}=$ $(\mathcal{C}(A A A C))^{+}, \quad(A C A G)^{+}$and $(C T G T)^{+}=(\mathcal{C}(A C A G))^{+},(A C A T)^{+}$and $(A T G T)^{+}=(\mathcal{C}(A C A T))^{+},(A A T G)^{+}$and $(A T T C)^{+}=\left(\mathcal{C}\left(\mathcal{P}^{3}(A A T G)\right)\right)$, and $(A G G G)^{+}$and $(C C C T)^{+}=(\mathcal{C}(A G G G))^{+}$(Figure 4.23). The repeats $(C C G G)^{+}$ (note that $\mathcal{C}(C C G G)=C C G G)$, and $(C C C G)^{+}$and $(C G G G)^{+}=(\mathcal{C}(C C C G))^{+}$ are almost absent (results not shown). A repeated tetranucleotide tetra ${ }^{+}$and its complementary repeated tetranucleotide $(\mathcal{C}(\text { tri }))^{+}$also have the same occurrences in the eukaryotic genomes. The repeated tetranucleotides tetra ${ }^{+}$and $(\mathcal{C}(\text { tetra }))^{+}$ also have increasing occurrences conversely to their number of hydrogen bonds, from the highest occurrences for the two repeats $(A A A T)^{+}$and $(A T T T)^{+}$with a total of eight hydrogen bonds to the lowest occurrences for the two repeats


Figure 4.23: Occurrence number $N\left(\right.$ tetra $\left.^{+}\right)$(Equation 3.13) (descending order, showing first 25) of the repeated tetranucleotides tetra $^{n}$ of length $l=4 n \geq 28$ nucleotides ( $n \geq 10$ ) in the eukaryotic genomes. The repeated tetranucleotides are generated from the unitary circular codes of tetranucleotides.
$(C C C G)^{+}$and $(C G G G)^{+}$with a total of 12 hydrogen bonds.

### 4.4.4 LaRgest repeated motifs in eukaryotic genomes

Table 4.32 shows the largest nucleotide lengths $l=2 n$ for the six repeated dinucleotides $d i^{n}, l=3 n$ for the 20 repeated trinucleotides $\operatorname{tr} i^{n}$ and $l=4 n$ for the 10 largest repeated tetranucleotides tri $^{n}$ in the eukaryotic genomes. The largest repeated dinucleotide $(A T)^{n}$ of length $l=11254$ nucleotides is observed in the chromosome 1 of Medicago truncatula. The largest repeated trinucleotide $(A T T)^{n}$ of length $l=19275$ nucleotides is found in the chromosome 9 of Citrus sinensis. The largest repeated tetranucleotide $(A T C C)^{n}$ of length $l=6952$ nucleotides is present in the chromosome 11 of Solanum pennellii.


Figure 4.24: Base number $N\left(\right.$ tetra $\left.^{+}\right)$(Equation 3.14) (descending order, showing first 25) of the repeated tetranucleotides tetra ${ }^{n}$ of length $l=4 n \geq 28$ nucleotides ( $n \geq 7$ ) in the eukaryotic genomes. The repeated tetranucleotides are generated from the unitary circular codes of tetranucleotides.

Table 4.32: Largest repeated motifs in nucleotides for the 6 repeated dinucleotides $d i^{n}$, for the 20 repeated trinucleotides $t r i^{n}$ and for the 10 largest repeated tetranucleotides $t e t r a^{n}$ in the eukaryotic genomes. The 1st and 5th columns indicate the unitary circular code (UCC) motif, the 2nd and 6th columns mention the genome $\mathcal{G}$ while its chromosome number $\mathcal{G}_{C h r}$ are found in 3rd and 7th columns, respectively, and the 4th and 8th columns gives the length in nucleotides of the UCC motif.

| UCC <br> motif | Genome $\mathcal{G}$ | $\mathcal{G}_{c h r}$ | Length (in bases) | UCC motif | Genome $\mathcal{G}$ | $\mathcal{G}_{c h r}$ | Length (in bases) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{AC})^{n}$ | Citrus sinensis | 7 | 4500 | $(\mathrm{CCG})^{n}$ | Oryza brachyantha | 9 | 555 |
| $(\mathrm{AG})^{n}$ | Citrus sinensis | 3 | 7648 | $(\mathrm{CCT})^{n}$ | Ficedula albicollis | 14 | 1065 |
| $(\mathrm{AT})^{n}$ | Medicago truncatula | 1 | 11254 | $(\mathrm{CGG})^{n}$ | Oryza brachyantha | 7 | 210 |
| $(\mathrm{CG})^{n}$ | Cucumis sativus | 7 | 432 | $(\mathrm{CGT})^{n}$ | Solanum pennellii | 8 | 1815 |
| $(\mathrm{CT})^{n}$ | Citrus sinensis | 5 | 7232 | $(\mathrm{CTG})^{n}$ | Ficedula albicollis | 12 | 723 |
| $(\mathrm{GT})^{n}$ | Beta vulgaris subsp. vulgaris | 5 | 2680 | $(\mathrm{CTT})^{n}$ | Cicer arietinum | Ca6 | 4263 |
| $(\mathrm{AAC})^{n}$ | Solanum pennellii | 9 | 8655 | $(\mathrm{GGT})^{n}$ | Homo sapiens | 2 | 630 |
| $(\mathrm{AAG})^{n}$ | Solanum pennellii | 12 | 10536 | $(\mathrm{GTT})^{n}$ | Cicer arietinum | Ca8 | 4239 |
| $(\mathrm{AAT})^{n}$ | Citrus sinensis | 4 | 12951 | $\left(\right.$ ATCC) ${ }^{n}$ | Solanum pennellii | 11 | 6952 |
| $(\mathrm{ACC})^{n}$ | Oryza brachyantha | 11 | 363 | $(\mathrm{ATCT})^{n}$ | Solanum pennellii | 6 | 5492 |
| $(\mathrm{ACG})^{n}$ | Bombus terrestris | B04 | 144 | $\left(\right.$ ATGG) ${ }^{n}$ | Solanum pennellii | 9 | 5076 |
| $(\mathrm{ACT})^{n}$ | Solanum pennellii | 12 | 1728 | $(\mathrm{AGAT})^{n}$ | Solanum pennellii | 10 | 4904 |
| $(\mathrm{AGC})^{n}$ | Ficedula albicollis | 21 | 3555 | $(\mathrm{AAAG})^{n}$ | Ficedula albicollis | 1 | 4780 |
| $(\mathrm{AGG})^{n}$ | Ficedula albicollis | 6 | 822 | $\left(\right.$ ATTT) ${ }^{n}$ | Cynoglossus semilaevis | 5 | 4268 |
| $(\mathrm{AGT})^{n}$ | Zea mays | 10 | 1926 | $(\mathrm{CTTT})^{n}$ | Ficedula albicollis | 1 | 4200 |
| $(\mathrm{ATC})^{n}$ | Camelina sativa | 5 | 2145 | $(\mathrm{AAGG})^{n}$ | Ficedula albicollis | 15 | 4048 |
| $(\mathrm{ATG})^{n}$ | Citrus sinensis | 9 | 2076 | $(\mathrm{CCTT})^{n}{ }^{n}$ | Ficedula albicollis | 1 | 3668 |
| $(\mathrm{ATT})^{n}$ | Citrus sinensis | 9 | 19275 | $(\mathrm{AGTG})^{n}$ | Cicer arietinum | Ca 2 | 3036 |

Table 4.33: Correlation matrix of the base number $B\left(\right.$ tri $\left.^{+}\right)$(Equation 3.14 and Figure 4.24) of all the repeated trinucleotides $t r i^{n}$ of length $l=3 n \geq 30$ nucleotides ( $n \geq 10$ ) in the eukaryotic genomes. The highest correlation (in bold) is always between a repeated trinucleotide tri ${ }^{+}$and its complementary repeated trinucleotide ( $\mathcal{C}($ tri $\left.)\right)^{+}$(note that $(A T C)^{+}$and $(A T G)^{+}=\left(\mathcal{C}\left(\mathcal{P}^{2}(A T C)\right)\right)^{+}$, and $(A G C)^{+}$and $(C T G)^{+}=\left(\mathcal{C}\left(\mathcal{P}^{2}(A G C)\right)\right)^{+}$, details in Section 4.4.2). There is no significant correlation between a repeated trinucleotide $t r i^{+}$and the size of genomes as well as the count of $A, C, G, T$ and $G C$ content of genomes.

|  | Size | ount | C count | G count | T count | GC content | AAC | G | AAT | ACC | ACG | T | A | AGG | T | A |  |  |  |  |  |  | G | G | GG | GTT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Size | 1 | 0.11 | -0.11 | -0.11 | 0.11 | -0.11 | 0.45 | 0.43 | 0.17 | 0.3 | 30.08 | 0.38 | 0.41 | 0.35 | 0.38 | 0.37 | 0.38 | 0.17 | 0.56 | 0.35 | 0.5 | 0.07 | 0.4 | 40.43 | 0.32 | 0.46 |
| A count | 0.11 | 1 | -1 | -1 | 1 | -1 | 0.14 | 0.07 | 0.17 | 0 | $0-0.19$ | 0.05 | -0.01 | 0.02 | 0.05 | 0.08 | 0.1 | 0.16 | -0.03 | 0.02 | -0.03 | -0.14 | 0 | 00.06 | 0.01 | 0.12 |
| C count | -0.11 | -1 | 1 | 1 | -1 | 1 | -0.14 | -0.07 | -0.17 | 0 | $0 \quad 0.19$ | -0.05 | 0.01 | -0.02 | -0.05 | -0.08 | -0.1 | -0.16 | 0.02 | -0.02 | 0.03 | 0.14 |  | $0-0.06$ | -0.01 | -0.12 |
| G count | -0.11 | -1 | 1 | 1 | -1 | 1 | -0.14 | -0.07 | -0.17 | 0 | $0 \quad 0.19$ | -0.05 | 0.01 | -0.02 | -0.05 | -0.08 | -0.1 | -0.16 | 0.03 | -0.02 | 0.03 | 0.14 |  | 0-0.06 |  | -0.12 |
| T count | 0.11 | 1 | -1 | -1 | 1 | -1 | 0.14 | 0.07 | 0.17 | 0 | $0-0.19$ | 0.05 | -0.01 | 0.02 | 0.05 | 0.08 | 0.1 | 0.16 | -0.02 | 0.02 | -0.03 | -0.14 | 0 | $0 \quad 0.06$ | 0 | 0.12 |
| GC content | -0.11 | -1 | 1 | 1 | -1 | 1 | -0.14 | -0.07 | -0.17 | 0 | $0 \quad 0.19$ | -0.05 | 0.01 | -0.02 | -0.05 | -0.08 | -0.1 | -0.16 | 0.03 | -0.02 | 0.03 | 0.14 |  | 0-0.06 | 0 | -0.12 |
| AAC | 0.45 | 0.14 | -0.14 | -0.14 | 0.14 | -0.14 | 1 | 0.57 | 0.75 | 0.55 | 50.19 | 0.38 | 0.29 | 0.58 | 0.38 | 0.85 | 0.86 | 0.75 | 0.3 | 0.57 | 0.23 | 0.21 | 0.29 | 90.56 | 0.56 | 0.98 |
| AAG | 0.43 | 0.07 | -0.07 | -0.07 | 0.07 | -0.07 | 0.57 | 1 | 0.18 | 0.75 | $5 \quad 0.29$ | 0.56 | 0.36 | 0.92 | 0.58 | 0.57 | 0.59 | 0.19 | 0.5 | 0.92 | 0.43 | 0.25 | 0.37 | 71 | 0.77 | 0.57 |
| AAT | 0.17 | 0.17 | -0.17 | -0.17 | 0.17 | -0.17 | 0.75 | 0.18 | 1 | 0.11 | 10 | 0.27 | 0.04 | 0.2 | 0.27 | 0.77 | 0.76 | 1 | 0.04 | 0.2 | 0.02 | 0.01 | 0.04 | 40.18 | 0.1 | 0.77 |
| ACC | 0.3 | 0 | 0 | 0 | 0 | 0 | 0.55 | 0.75 | 0.11 | 1 | 10.44 | 0.43 | 0.49 | 0.72 | 0.43 | 0.53 | 0.56 | 0.11 | 0.38 | 0.7 | 0.26 | 0.38 | 0.5 | $5 \quad 0.74$ | 1 | 0.55 |
| ACG | 0.08 | -0.19 | 0.19 | 0.19 | -0.19 | 0.19 | 0.19 | 0.29 | 0 | 0.44 | 4 | 0.28 | 0.18 | 0.28 | 0.27 | 0.21 | 0.21 | 0 | 0.19 | 0.28 | 0.14 | 0.93 | 0.18 | 80.28 | 0.44 | 0.18 |
| ACT | 0.38 | 0.05 | -0.05 | -0.05 | 0.05 | -0.05 | 0.38 | 0.56 | 0.27 | 0.43 | 0.28 | 1 | 0.25 | 0.42 | 0.99 | 0.51 | 0.52 | 0.27 | 0.25 | 0.42 | 0.19 | 0.21 | 0.25 | 50.57 | 0.44 | 0.4 |
| AGC | 0.41 | -0.01 | 0.01 | 0.01 | -0.01 | 0.01 | 0.29 | 0.36 | 0.04 | 0.49 | 9.18 | 0.25 | 1 | 0.37 | 0.25 | 0.3 | 0.32 | 0.04 | 0.21 | 0.36 | 0.15 | 0.15 | 1 | 10.37 | 0.49 | 0.29 |
| AGG | 0.35 | 0.02 | -0.02 | -0.02 | 0.02 | -0.02 | 0.58 | 0.92 | 0.2 | 0.72 | 20.28 | 0.42 | 0.37 | 1 | 0.43 | 0.5 | 0.51 | 0.2 | 0.39 | 1 | 0.32 | 0.22 | 0.38 | 80.93 | 0.73 | 0.58 |
| AGT | 0.38 | 0.05 | -0.05 | -0.05 | 0.05 | -0.05 | 0.38 | 0.58 | 0.27 | 0.43 | 3.27 | 0.99 | 0.25 | 0.43 | 1 | 0.52 | 0.53 | 0.27 | 0.25 | 0.43 | 0.18 | 0.21 | 0.25 | 50.58 | 0.44 | 0.39 |
| ATC | 0.37 | 0.08 | -0.08 | -0.08 | 0.08 | -0.08 | 0.85 | 0.57 | 0.77 | 0.53 | 30.21 | 0.51 | 0.3 | 0.5 | 0.52 | 1 | 0.99 | 0.77 | 0.28 | 0.49 | 0.21 | 0.19 | 0.31 | 10.56 | 0.54 | 0.87 |
| ATG | 0.38 | 0.1 | -0.1 | -0.1 | 0.1 | -0.1 | 0.86 | 0.59 | 0.76 | 0.56 | 60.21 | 0.52 | 0.32 | 0.51 | 0.53 | 0.99 | 1 | 0.76 | 0.28 | 0.51 | 0.21 | 0.22 | 0.32 | 2.58 | 0.56 | 0.88 |
| ATT | 0.17 | 0.16 | -0.16 | -0.16 | 0.16 | -0.16 | 0.75 | 0.19 | 1 | 0.11 | 10 | 0.27 | 0.04 | 0.2 | 0.27 | 0.77 | 0.76 | 1 | 0.04 | 0.2 | 0.02 | 0.01 | 0.04 | 40.18 | 0.11 | 0.78 |
| CCG | 0.56 | -0.03 | 0.02 | 0.03 | -0.02 | 0.03 | 0.3 | 0.5 | 0.04 | 0.38 | 0.19 | 0.25 | 0.21 | 0.39 | 0.25 | 0.28 | 0.28 | 0.04 | 1 | 0.39 | 0.95 | 0.17 | 0.22 | 2.5 | 0.39 | 0.31 |
| CCT | 0.35 | 0.02 | -0.02 | -0.02 | 0.02 | -0.02 | 0.57 | 0.92 | 0.2 | 0.7 | $7 \quad 0.28$ | 0.42 | 0.36 | 1 | 0.43 | 0.49 | 0.51 | 0.2 | 0.39 | 1 | 0.32 | 0.22 | 0.37 | $7 \quad 0.93$ | 0.72 | 0.58 |
| CGG | 0.5 | -0.03 | 0.03 | 0.03 | -0.03 | 0.03 | 0.23 | 0.43 | 0.02 | 0.26 | $6 \quad 0.14$ | 0.19 | 0.15 | 0.32 | 0.18 | 0.21 | 0.21 | 0.02 | 0.95 | 0.32 | 1 | 0.13 | 0.16 | $6 \quad 0.42$ | 0.27 | 0.24 |
| CGT | 0.07 | -0.14 | 0.14 | 0.14 | -0.14 | 0.14 | 0.21 | 0.25 | 0.01 | 0.38 | 0.93 | 0.21 | 0.15 | 0.22 | 0.21 | 0.19 | 0.22 | 0.01 | 0.17 | 0.22 | 0.13 | 1 | 0.16 | $6 \quad 0.24$ | 0.38 | 0.16 |
| CTG | 0.4 | 0 | 0 | 0 | 0 | 0 | 0.29 | 0.37 | 0.04 | 0.5 | 50.18 | 0.25 | 1 | 0.38 | 0.25 | 0.31 | 0.32 | 0.04 | 0.22 | 0.37 | 0.16 | 0.16 | 1 | $1 \quad 0.37$ | 0.5 | 0.3 |
| CTT | 0.43 | 0.06 | -0.06 | -0.06 | 0.06 | -0.06 | 0.56 | 1 | 0.18 | 0.74 | 40.28 | 0.57 | 0.37 | 0.93 | 0.58 | 0.56 | 0.58 | 0.18 | 0.5 | 0.93 | 0.42 | 0.24 | 0.37 | 71 | 0.76 | 0.56 |
| GGT | 0.32 | 0.01 | -0.01 | 0 | 0 |  | 0.56 | 0.77 | 0.1 | 1 | 10.44 | 0.44 | 0.49 | 0.73 | 0.44 | 0.54 | 0.56 | 0.11 | 0.39 | 0.72 | 0.27 | 0.38 | 0.5 | $5 \quad 0.76$ | 1 | 0.55 |
| GTT | 0.46 | 0.12 | -0.12 | -0.12 | 0.12 | -0.12 | 0.98 | 0.57 | 0.77 | 0.55 | 50.18 | 0.4 | 0.29 | 0.58 | 0.39 | 0.87 | 0.88 | 0.78 | 0.31 | 0.58 | 0.24 | 0.16 | 0.3 | $3 \quad 0.56$ | 0.55 | 1 |

### 4.4.5 SCARCITY OF REPEATED TRINUCLEOTIDES IN EUKARYOTIC GENOMES

Table 4.34: Scarcity of repeated trinucleotides ( $\mathrm{Tri}^{+}$motifs) in the large genomes of eukaryotes. The 1st and 5th columns mention the 59 eukaryotic genomes $\mathcal{G}$ of large sizes $N(\mathcal{G})>300000 \mathrm{~kb}$, the 2nd 6th, 3th and 7th, and 4th and 8th provide the ratios $r\left(D i^{+}, \mathcal{G}\right)(\%), r\left(\operatorname{Tri}^{+}, \mathcal{G}\right)(\%)$ and $r\left(\right.$ Tetra $\left.^{+}, \mathcal{G}\right)(\%)$, respectively, (Equation 3.16) giving the proportion of the total base numbers $B\left(D \imath^{+}, \mathcal{G}\right), B\left(T r i^{+}, \mathcal{G}\right)$ and $B\left(\right.$ Tetra $\left.^{+}, \mathcal{G}\right)$, respectively, (Equation 3.14) of all the repeated dinucleotides $D i^{+}$(Section 2.3.2), all the repeated trinucleotides $\mathrm{Tr}^{+}$(Section 2.3.2) and all the repeated tetranucleotides $\mathrm{Tetra}^{+}$(Section 2.3.2), respectively, in the large eukaryotic genomes $\mathcal{G}$. The means $\bar{r}\left(D i^{+}\right), \bar{r}\left(T r i^{+}\right)$and $\bar{r}\left(T e t r a{ }^{+}\right)$(Equation 3.17) and the medians $\tilde{r}\left(D i^{+}\right), \tilde{r}\left(\operatorname{Tri}^{+}\right)$and $\tilde{r}\left(\operatorname{Tetra}^{+}\right)$of the ratios $r\left(D i^{+}, \mathcal{G}\right), r\left(\operatorname{Tri}^{+}, \mathcal{G}\right)$ and $r\left(\operatorname{Tetra}^{+}, \mathcal{G}\right)$ in the genomes of eukaryotes $\mathbb{E}$ lead to Equation 4.1.

| Genome $\mathcal{G}$ | $r\left(D i^{+}, \mathcal{G}\right)$ | $r(T r i+, \mathcal{G})$ | $r\left(\right.$ Tetra $\left.^{+}, \mathcal{G}\right)$ | Genome $\mathcal{G}$ | $r\left(D i^{+}, \mathcal{G}\right)$ | $r(T r i+, \mathcal{G})$ | $r\left(\right.$ Tetra $\left.^{+}, \mathcal{G}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anolis carolinensis | 0.814 | 2.602 | 0.56 | Microtus ochrogaster | 3.528 | 0.319 | 1.483 |
| Beta vulgaris | 0.827 | 0.668 | 0.05 | Monodelphis domestica | 2.447 | 0.256 | 2.066 |
| Bos taurus | 0.45 | 0.022 | 0.008 | Mus musculus | 5.061 | 0.812 | 2.51 |
| Brassica napus | 0.332 | 0.072 | 0.013 | Nomascus leucogenys | 0.541 | 0.066 | 0.494 |
| Brassica oleracea | 0.459 | 0.093 | 0.011 | Oreochromis niloticus | 1.684 | 0.125 | 0.292 |
| Callithrix jacchus | 0.791 | 0.033 | 0.391 | Ornithorhynchus anatinus | 0.223 | 0.09 | 0.262 |
| Camelina sativa | 0.707 | 0.202 | 0.009 | Oryctolagus cuniculus | 1.086 | 0.044 | 0.378 |
| Canis lupus familiaris | 1.119 | 0.207 | 1.734 | Oryza sativa Japonica | 0.859 | 0.133 | 0.069 |
| Capra hircus | 0.458 | 0.041 | 0.027 | Oryzias latipes | 0.188 | 0.061 | 0.827 |
| Chlorocebus sabaeus | 0.524 | 0.106 | 0.87 | Ovis aries | 0.506 | 0.06 | 0.033 |
| Chrysemys picta bellii | 0.565 | 0.007 | 0.091 | Pan paniscus | 0.503 | 0.07 | 0.355 |
| Cicer arietinum | 0.948 | 1.285 | 0.17 | Pan troglodytes | 0.501 | 0.072 | 0.371 |
| Cynoglossus semilaevis | 1.297 | 0.613 | 0.52 | Papio anubis | 0.462 | 0.082 | 0.774 |
| Danio rerio | 8.289 | 0.987 | 3.884 | Phaseolus vulgaris | 0.571 | 0.134 | 0.005 |
| Elaeis guineensis | 0.88 | 0.096 | 0.054 | Poecilia reticulata | 1.296 | 0.413 | 1.182 |
| Equus caballus | 0.18 | 0.012 | 0.057 | Pongo abelii | 0.432 | 0.068 | 0.32 |
| Esox lucius | 1.028 | 0.022 | 0.056 | Populus trichocarpa | 1.289 | 0.21 | 0.018 |
| Felis catus | 2.624 | 0.152 | 1.027 | Rattus norvegicus | 5.972 | 0.57 | 1.423 |
| Ficedula albicollis | 0.201 | 0.389 | 1.783 | Salmo salar | 6.069 | 0.089 | 1.268 |
| Gallus gallus | 0.07 | 0.029 | 0.323 | Setaria italica | 0.225 | 0.033 | 0.022 |
| Glycine max | 2.034 | 0.26 | 0.013 | Solanum lycopersicum | 0.878 | 0.176 | 0.034 |
| Gorilla gorilla gorilla | 0.448 | 0.055 | 0.315 | Solanum pennellii | 0.634 | 0.331 | 0.135 |
| Gossypium raimondii | 0.24 | 0.199 | 0.044 | Sorghum bicolor | 0.577 | 0.204 | 0.058 |
| Homo sapiens | 0.713 | 0.086 | 0.502 | Sus scrofa | 0.779 | 0.048 | 0.455 |
| Lepisosteus oculatus | 0.051 | 0.325 | 0.028 | Taeniopygia guttata | 0.124 | 0.079 | 0.214 |
| Macaca fascicularis | 0.612 | 0.109 | 0.979 | Theobroma cacao | 0.562 | 0.048 | 0.011 |
| Macaca mulatta | 0.595 | 0.093 | 0.778 | Vigna radiata | 3.389 | 0.251 | 0.023 |
| Malus domestica | 0.9 | 0.056 | 0.023 | Vitis vinifera | 0.987 | 0.284 | 0.035 |
| Medicago truncatula | 1.206 | 0.156 | 0.014 | Zea mays | 0.17 | 0.038 | 0.008 |
| Meleagris gallopavo | 0.088 | 0.035 | 0.162 | Means $\bar{r}$ | 1.203 | 0.24 | 0.502 |
|  |  |  |  | Medians $\tilde{r}$ | 0.634 | 0.096 | 0.214 |

Table 4.34 shows the ratios $r\left(D i^{+}, \mathcal{G}\right), r\left(\operatorname{Tri}^{+}, \mathcal{G}\right)$ and $r\left(\right.$ Tetra $\left.^{+}, \mathcal{G}\right)$ giving the proportion of the total base numbers $B\left(D i^{+}, \mathcal{G}\right), B\left(T r i^{+}, \mathcal{G}\right)$ and $B\left(\right.$ Tetra $\left.^{+}, \mathcal{G}\right)$ of all the repeated dinucleotides $D \imath^{+}$(Section 2.3.2), all the repeated trinucleotides $\mathrm{Tr}^{+}$(Section 2.3.2) and all the repeated tetranucleotides $\mathrm{Tetra}^{+}$(Section 2.3.2) in the 59 large eukaryotic genomes $\mathcal{G}$ (sizes $N(\mathcal{G})>300000 \mathrm{~kb})$. Interestingly, the means $\bar{r}\left(D^{+}\right), \bar{r}\left(T r i^{+}\right)$and $\bar{r}\left(\right.$ Tetra $\left.^{+}\right)$(Equation 3.17) and the medians $\tilde{r}\left(D i^{+}\right)$, $\tilde{r}\left(\right.$ Tri $\left.^{+}\right)$and $\tilde{r}\left(\right.$ Tetra $\left.^{+}\right)$of the ratios $r\left(D \imath^{+}, \mathcal{G}\right), r\left(\operatorname{Tri}^{+}, \mathcal{G}\right)$ and $r\left(\right.$ Tetra $\left.^{+}, \mathcal{G}\right)$, re-
spectively, in the genomes of eukaryotes $\mathbb{E}$ leads both to the same following result

$$
\left\{\begin{array}{l}
\bar{r}\left(D i^{+}\right)>\bar{r}\left(\text { Tetra }^{+}\right)>\bar{r}\left(\text { Tri }^{+}\right)  \tag{4.1}\\
\tilde{r}\left(D i^{+}\right)>\tilde{r}\left(\text { Tetra }^{+}\right)>\tilde{r}\left(\text { Tri }^{+}\right)
\end{array}\right.
$$

These inequalities are evaluated by two statistical tests: a paired sample Student's t-test (parametric statistical hypothesis test assuming a normal distribution of the population) and a Wilcoxon signed-rank W-test (non-parametric statistical hypothesis test). The comparisons of the means $\bar{r}\left(D i^{+}\right)$and $\bar{r}\left(T e t r a{ }^{+}\right)$, and the means $\bar{r}\left(\right.$ Tetra $\left.^{+}\right)$and $\bar{r}\left(T r i^{+}\right)$with the t -test have significant $p$-values equal to $3 \times 10^{-5}$ and $7 \times 10^{-3}$, respectively. The comparisons of the distribution of $\mathrm{Di}^{+}$and Tetra+, and the distribution of $\mathrm{Tetra}^{+}$and $\mathrm{Tri}^{+}$with the Wilcoxon test also have significant $p$-values equal to $10^{-6}$ and $9 \times 10^{-3}$, respectively. Thus, the total base number of $D i^{+}$is greater than the total base number of Tetra ${ }^{+}$ which is greater than the total base number of $\mathrm{Tri}^{+}$. In other words, there is a scarcity of repeated trinucleotides in the large eukaryotic genomes compared to the repeated dinucleotides and the repeated tetranucleotides. For the eukaryotic genomes $\mathcal{G}$ of small sizes $N(\mathcal{G})<300000 \mathrm{~kb}$, the analysis has the same statistical trend. However, it is not conclusive and should be investigated in the future with the increase of genome data.

### 4.5 Identical trinucleotide pairs of the $X$ Circular code in euKaryotic gene sequences

Unitary circular codes (UCC) of dinucleotides, trinucleotides and tetranucleotides are associated with the repeated dinucleotides ( $D i^{+}$motifs), the repeated trinucleotides ( $t^{+}$motifs) and the repeated tetranucleotides ( Tetra $^{+}$motifs) which are identified in the genomes of eukaryotes. Furthermore, there is a scarcity of $t^{+}$motifs in the large genomes of eukaryotes compared to the $\mathrm{Di}^{+}$and Tetra ${ }^{+}$motifs (Section 4.4.5). Otherwise, a circular code $X$ is observed in genes of bacteria, eukaryotes, plasmids and viruses (Arquès and Michel, 1996; Michel, 2015)). The problem investigated here is whether the unitary circular codes of trinucleotides in genomes may have some traces in the trinucleotide circular code $X$ in genes.


Figure 4.25: Identical trinucleotide pairs of the maximal $C^{3}$ self-complementary trinucleotide circular code $X$ preferentially used in the gene sequences of eukaryotes identified by the median $\tilde{r}\left(t_{1} t_{2}\right)$ (Equation 3.27) of the observed/theoretical ratios $r\left(t_{1} t_{2}, \mathcal{G}\right)$ (Equation 3.26) of the trinucleotide pairs $t_{1} t_{2} \in X^{2}$. Each figure gives in ordinate the median $\tilde{r}\left(t_{1} t_{2}\right)$ of a given trinucleotide $t_{1} \in X$ (in label) by varying the 20 trinucleotides $t_{2} \in X$ in abscissa.

Figure 4.25 identifies a new property of the circular code $X$. Indeed, by varying the 20 trinucleotides $t_{2} \in X$ for a given trinucleotide $t_{1} \in X$, the medians $\tilde{r}\left(t_{1} t_{2}\right)$ (Equation 3.27) of the observed/theoretical ratios $r\left(t_{1} t_{2}, \mathcal{G}\right)$ (Equation 3.26) of the trinucleotide pairs $t_{1} t_{2} \in X^{2}$ identify 14 trinucleotide pairs such that the values of $\tilde{r}\left(t_{1} t_{2}\right)$ are maximal when the trinucleotide $t_{2}=t_{1}$. These 14 trinucleotide pairs $t t$ with an identical trinucleotide $t$ are described according to $t$ as follows:

$$
\begin{align*}
& t \in X^{\prime}=\{A A C, A C C, A T C, C A G, C T C, C T G, G A A  \tag{4.2}\\
&G A G, G A T, G G T, G T A, G T T, T A C, T T C\}
\end{align*}
$$

where $X^{\prime}$ is a subset of $X$. There $\tilde{r}(t t)$ values are significantly greater than 1 (see Remark 2): $\tilde{r}(A A C A A C)=1.70, \tilde{r}(A C C A C C)=2.26$, $\tilde{r}(A T C A T C)=1.93, \tilde{r}(C A G C A G)=1.79, \tilde{r}(C T C C T C)=1.90$, $\tilde{r}(C T G C T G)=1.71, \tilde{r}(G A A G A A)=2.03, \tilde{r}(G A G G A G)=1.98$, $\tilde{r}(G A T G A T)=1.93, \tilde{r}(G G T G G T)=2.25, \tilde{r}(G T A G T A)=1.57$, $\tilde{r}(G T T G T T)=1.69, \tilde{r}(T A C T A C)=1.65$ and $\tilde{r}(T T C T T C)=1.99$. The trinucleotide pair GGCGGC has the highest value ( $\tilde{r}(G G C G G C)=1.33)$ still close to 1 . The exceptions are the five trinucleotide pairs $t_{1} t_{2} \in\{A A T G T C, A T T G T T, G A C A T T, G C C A C C, G T C A T C\}$ with $\tilde{r}\left(t_{1} t_{2}\right)$ values close to 1 (except one case): $\tilde{r}(A A T G T C)=1.40, \tilde{r}(A T T G T T)=1.32$, $\tilde{r}(G A C A T T)=1.39, \tilde{r}(G C C A C C)=1.83$ and $\tilde{r}(G T C A T C)=1.59$.

Surprisingly, the trinucleotide set $X^{\prime}$ is also self-complementary, i.e. $X^{\prime}=$ $\mathcal{C}\left(X^{\prime}\right)$. All these results are retrieved with the two other ratios $r\left(t_{1} t_{2}\right)$ (Equation 3.26) and $\bar{r}\left(t_{1} t_{2}\right)$ (Equation 3.27) (results not shown). Thus, with a few exceptions related to values of $\tilde{r}\left(t_{1} t_{2}\right), r\left(t_{1} t_{2}\right)$ and $\bar{r}\left(t_{1} t_{2}\right)$ close to 1 , identical trinucleotide pairs of the circular code $X$ are preferentially used in the genes of eukaryotes.

### 4.6 Summary

New properties of the circular code theory is identified here. We were able to show strong evidence in favour of the $X$ circular code. The first was the presence of the conserved G530, A1492 and A1493 nucleotides of the ribosomal rRNA in $X$ circular code motifs. These nucleotides were proven to have a function in the translation process by distinguishing cognate from non-cognate tRNAs by anticodon-codon interactions with the mRNA codon (Wilson, 2014). Our study in the ribosome was conducted in an expansive manner, we started in the decoding center, then we moved to conserved $X$ circular code motifs near the decoding center. Afterwards, we showed an exhaustive study of $X$ circular code motifs in tRNA sequences for which we presented a coverage of $X$ motifs in tRNA sequences.

The study on the eukaryotic genomes showed us that the $X_{0}$ circular code is preferential when compared to $X_{1}$ and $X_{2}, 23$ bijective circular code transformations and 30 random generated codes by analysing the occurrence of the large motifs retrieved from the mentioned codes. The results presented distinguish the $X$ circular code. These studies were done on the genomes of 138 organisms with complete chromosomes, while giving a detailed study for the Homo sapiens genome. We also saw the preferential presence of $X$ circular code motifs in coding regions when compared to non-coding regions in eukaryotic genomes.

We were able to find interesting facts about the scarcity of trinucleotide simple repeats when compared to its dinucleotide and tetranucleotide counter parts. While showing there is no significant correlation between a repeated trinucleotide and the size of genomes as well as the count of $A, C, G, T$ and $G C$ content of genomes. We found that there is a preference for identical trinucleotide pairs in the gene sequences of eukaryotic genomes when it comes to $X$ circular code trinucleotides.

In the next chapter we will conclude our work with a summary of our finding and some hypothesises that can be drawn from these results while also proposing some theories and questions raised.

## 5

## Conclusion

The results here obtained in this thesis bring several new contributions to the circular code theory. This is the first time that $X$ circular code motifs are studied in a biological context. Thus, these motifs have the circular code property (Definition 2.5), allowing retrieval of the reading frame, the property $C^{3}$ (Definition 2.10) allowing retrieval of the two shifted frames and the complementary property (Definition 2.9) allowing pairing between $X$ circular code motifs. $X$ circular code motifs were found in the sequences of ribosomes (mRNA, tRNA and rRNA) and in complete sequences of eukaryotic chromosomes. The results strengthen the concept of translation code based on the circular code proposed in Michel, 2012.

The circular code theory contributed to the analysis of the ribosome decoding center, in particular to its primary structure which is related to the mathematical property of circular code. The universally conserved nucleotides A1492 and A1493 in all studied rRNAs of bacteria, archaea, nuclear eukaryotes, and chloroplasts belongs to $X$ motifs ( $m_{A A}$, Table 4.1). The conserved nucleotide G530 in rRNAs of bacteria and archaea also belongs to $X$ motifs ( $m_{G}$, Table 4.2). The development of a tool (Section 3.4.1) associated with the global multiple sequence alignment allows to identify the $X$ motifs $m_{G}$ in nuclear and chloroplast rRNAs (Table 4.2) as it was not previously identified experimentally. Furthermore, it reveals a new $X$ motif ( $m$ ) which is universally conserved in the seven studied organisms (Table 4.3). Finally, the three $X$ motifs $m_{A A}, m_{G}$, and $m$ belong to the ribosome decoding center in all studied rRNAs of bacteria, archaea, nuclear eukaryotes, and chloroplasts (Figures 4.1, 4.2, 4.3, 4.4, 4.5, 4.6 and 4.7).

Several biological considerations can be stressed from these results. The function of the ribosome decoding center which has been attributed to a very few nucleotides only, precisely the nucleotides A1492, A1493 and G530, may
be related to motifs containing at least two successive trinucleotides up to a maximum of five successive trinucleotides (Tables 4.1 and 4.2).

Near the ribosome decoding center seven $X$ circular code motifs $\operatorname{Pr} R N A X m$ which are conserved in 16 S rRNAs of bacteria and archaea are identified (Figures 4.8, 4.9 and 4.10), in particular the large $X$ motif $\operatorname{Pr} R N A X m_{1}$ $G C, G G T, A A T, A C C, G G C, G G C, C$ of 18 nucleotides in $P$. furiosus, the large common $X$ motif $\operatorname{PrRNAX} m_{3} G, A A Y, R_{1} C C, G R_{2} T, G G C, G A A, G G C$ of 19 nucleotides in E. coli $\left(Y=T, R_{1}=A\right.$ and $\left.R_{2}=G\right)$ and T. thermophilus $\left(Y=C, R_{1}=G\right.$ and $\left.R_{2}=A\right)$, the large $X$ motif $\operatorname{Pr} R N A X m_{4}$ $T A, G A T, A C C, C T G, G T A, G T C, C A$ of 19 nucleotides in E. coli, the large common $X$ motif $\operatorname{Pr} R N A X m_{6} T, T A C, G R C, C W G, G G C, K A C, A C$ of $18 \mathrm{nu}-$ cleotides in $E$. coli $(R=A, W=A$ and $K=T)$ and $T$. thermophilus ( $R=G$, $W=T$ and $K=G)$.

Four $X$ circular code motifs $\operatorname{Er} R N A X m$ which are conserved in 18 S rRNAs of S. cerevisiae, T. aestivum and H. sapiens are found near the ribosome decoding center (Figures 4.11, 4.12, 4.13), in particular the large common $X$ motif $E r R N A X m_{2} G, N T C, G A A, G A Y, G A T, C A G, A T$ of 18 nucleotides in $S$. cerevisiae, T. aestivum and H. sapiens and the large $X$ motif $\operatorname{Er} R N A X m_{4}$ $T C, T T C, A A C, G A G, G A A, T T C, C T$ of 19 nucleotides in $S$. cerevisiae.

These $X$ circular code motifs from the rRNA could hold some function in the positioning of the tRNA during the translation process by interacting with the $X$ motif from the tRNA sequence.

The final step of the ribosomal study was to expand it to include the tRNA sequences of E. coli, T. thermophilus and P. furiosus which showed several new features for the structure of tRNAs. The high coverage of $X$ motifs in the 5 ' and 3 ' regions of these tRNAs ( $88 \%$ and $71 \%$, respectively; with the exception of the 3 ' regions of tRNAs of Asp, Glu, His and SeC; Table 4.27) means that tRNAs may be constructed by a concatenation of $X$ motifs. This hypothesis is strengthened by the fact that four very large X motifs of length greater than or equal to 20 nucleotides are found in tRNAs having in average 79 nucleotides (Table 4.27): Ala - tRNAXm $\quad$ GC, CTC, AAT, GGC, ATT, GAG, GAG, GTC, A of 24 nucleotides in $\quad$. thermophilus, Glu - tRNAXm ${ }_{2}$ $T A, G A G, G C C, C A G, G A C, A C C, G C C, C T$ of 22 nucleotides in $E$. coli, Ser - tRNAXm $m_{3} T G, G T C, G A A, G G C, G G C, A C C, C T G, C T$ of 22 nucleotides in $T$. thermophilus and Tyr $-t R N A X m_{2}$ $T, G C C, G T C, A T C, G A C, T T C, G A A, G G T, T$ of 23 nucleotides in E. coli, and 14 large $X$ motifs of lengths 16-19 nucleotides: $\operatorname{Arg}-t R N A X m_{1}$, $\operatorname{Arg}-t R N A X m_{3}, \operatorname{Arg}-t R N A X m_{4}, \operatorname{Arg}-t R N A X m_{6}, A s n-t R N A X m_{3}$, $A s p-t R N A X m_{2}$, Ile $-t R N A X m_{2}, L e u-t R N A X m_{1}, S e r-t R N A X m_{2}$, $S e r-t R N A X m_{4}, S e r-t R N A X m_{5}, S e r-t R N A X m_{7}, T h r-t R N A X m_{4}$ and Val - $t R N A X m_{3}$. Remember that $X$ motifs of lengths equal to 9 nucleotides retrieve the reading frame with a probability of $99.9 \%$ and $X$ motifs of lengths
greater than or equal to 12 nucleotides retrieve, by definition, the reading frame with a probability of $100 \%$ (Table 3 and Fig. 4 in (Michel, 2012)). We also note that the coverage and the length of the $X$ motifs could be greater if substitutions in $X$ motifs were considered.

New properties of this circular code theory are also identified here with statistical studies of $X$ large motifs in genomes of eukaryotes. This study shines light on non-gene regions, that were not examined previously, as well as gene regions. It has also been proposed that the circular code $X$, which is associated with the regular RNA transcription, may use its bijective transformation codes for coding nucleotide exchanging RNA transcription particularly in mitochondria (Michel and Seligmann, 2014). The large $X$ motifs (having lengths $l \geq 15$ trinucleotides and cardinalities (composition) Card $\geq 10$ trinucleotides, Equation 3.4) have the highest occurrence in genomes of eukaryotes compared to (i) its 23 large bijective motifs from the bijective transformation circular codes, (ii) its two large permuted motifs $m\left(X_{1}\right)$ and $m\left(X_{2}\right)$ from the permuted circular codes $X_{1}=\mathcal{P}(X)$ and $X_{2}=\mathcal{P}^{2}(X)$, and (iii) large random motifs $m(R)$ from random codes $R$ (Section 4.3.1). The largest $X$ motifs identified in genomes are presented (Section 4.3.2 Table 4.28), e.g. an $X$ motif in a non-gene region of the genome Solanum pennellii with a length of 155 trinucleotides ( 465 nucleotides) and an expectation $E=10^{-71}$ (Equation 3.6), two $X$ motifs in non-gene regions of the genome Salmo salar with lengths of 118 trinucleotides (354 nucleotides) and an expectation $E=10^{-52}$, etc. Large $X$ motifs are also found in the human genome (Section 4.3.4 and Table 4.29). The largest $X$ motif occurs in a non-gene region of the human chromosome 13 with a length of 36 trinucleotides and an expectation $E=10^{-11}$.
$X$ motifs in non-gene regions of genomes are possibly evolutionary relics of primitive genes using the circular code for translation. However, the mean value $\bar{r}_{m(X)}(\mathcal{G})$ and the median value $\tilde{r}_{m(X)}(\mathcal{G})$ giving the proportion of $X$ motifs (having lengths $l \geq 10$ trinucleotides and cardinalities Card $\geq 5$ trinucleotides in genes/non-genes of the 138 complete eukaryotic genomes $\mathcal{G}$ are close to 8 $\left(\bar{r}_{m(X)}(\mathcal{G})=9.3 \approx \tilde{r}_{m(X)}(\mathcal{G})=7.6 \approx 8\right.$, Section 4.3 .5 and Table 4.30). This factor of 8 is retrieved for the $X$ motifs in genes/non-genes of the 24 human chromosomes $\mathcal{H}_{C h r}\left(\bar{r}_{m(X)}\left(\mathcal{H}_{C h r}\right)=8.1 \approx \tilde{r}_{m(X)}\left(\mathcal{H}_{C h r}\right)=7.8 \approx 8\right.$, Section 4.3.6 and Table 4.31).

Therefore, the $X$ motifs is found in non-coding regions but occur preferentially in coding region. This property is true whatever the base content of genes in the genomes as there is no correlation between the base proportion of genes/non-genes in genomes and the base proportion of $X$ motifs in genes/nongenes of genomes (Figures 4.14, 4.16). From a biological point of view, this property may be explained by the fact that mutations (substitution, insertion and deletion of nucleotides) are more frequent in non-gene regions compared to genes. Finally, the statistical analysis developed here is based on the search of exact $X$ motifs. $X$ motifs with a few mutations in genomes of eukaryotes
should also be investigated in future.

The origin of this trinucleotide circular code $X$ in genes is an open problem since its discovery in 1996. We show in our work that the circular code concept in low-complexity DNA regions exists with the unitary circular codes $(U C C)$ of dinucleotides, trinucleotides and tetranucleotides generating $U C C$ motifs of repeated dinucleotides ( $\mathrm{i}^{+}$motifs), repeated trinucleotides ( $\mathrm{Tri}^{+}$ motifs) and repeated tetranucleotides (Tetra ${ }^{+}$motifs) in eukaryotic genomes. Precisely, $12 U C C$ codes of dinucleotides are all strong comma-free, and four of them $\{A T\},\{C G\},\{G C\}$ and $\{T A\}$ are in addition self-complementary. 48 $U C C$ codes of trinucleotides are strong comma-free and $12 U C C$ codes of trinucleotides are comma-free. 180 UCC codes of tetranucleotides are strong commafree, $60 U C C$ codes of tetranucleotides are comma-free and 12 strong commafree $\{A A T T\},\{A C G T\},\{A G C T\},\{C A T G\},\{C C G G\},\{C T A G\},\{G A T C\}$, $\{G G C C\},\{G T A C\},\{T C G A\},\{T G C A\}$ and $\{T T A A\}$ are in addition selfcomplementary. Thus, the $D i^{+}$, Tri $^{+}$and Tetra ${ }^{+}$motifs allow to retrieve, to main and to synchronize a frame modulo 2 , modulo 3 and modulo 4, respectively, in non-coding regions. Furthermore, the $C^{2}, C^{3}$ and $C^{4}$ properties allow to retrieve, maintain and synchronize the shifted frames and the self-complementary property allows DNA pairing in non coding-regions. An $U C C$ motif and its complementary $U C C$ motif have the same distribution in eukaryotic genomes, both from their occurrence number and their total base number. This property is observed with the $\mathrm{Di}^{+}, \mathrm{Tri}^{+}$and $\mathrm{Tetra}^{+}$motifs. In addition for the $\mathrm{Tri}^{+}$and Tetra ${ }^{+}$motifs, an $U C C$ motif and its complementary $U C C$ motif have increasing occurrences conversely to their number of hydrogen bonds. For the $D i^{+}$motifs, the repeat $(C G)^{+}$has indeed the lowest occurrence but the repeat $(A T)^{+}$does not have the highest occurrence. The largest nucleotide lengths of $\mathrm{Di}^{+}, \mathrm{Tri}^{+}$ and Tetra ${ }^{+}$motifs in the studied eukaryotic genomes are given in Table 4.32.

Surprisingly, a scarcity of repeated trinucleotides ( $T r r i^{+}$motifs) in the large eukaryotic genomes is observed compared to the the $\mathrm{Di}^{+}$and Tetra ${ }^{+}$motifs. This statistical result is found with the mean and the median and confirmed by two statistical tests (a paired sample Student's $t$-test and a Wilcoxon signed-rank $W$-test). Thus, the unitary circular codes of trinucleotides associated to the repeated trinucleotides in eukaryotic genomes may have been involved in the formation of the trinucleotide circular code $X$ in genes. This is mainly due to the fact that unitary circular code motifs are a circular code of cardinality equal to 1 , one-point mutation would make this motif a circular code motif of cardinality. So on and so forth, we can easily have a circular code of cardinality 5 by having that same number as point mutations. A class of circular code motifs we found abundantly in the eukaryotic genomes.

A consequence of such an assumption would be the persistence of some statistical properties of repeated trinucleotides in the circular code $X$. Unexpectedly, identical trinucleotide pairs of the circular code $X$ are preferentially used in the
eukaryotic genes. Indeed, 14 trinucleotides among 20 of the circular code $X$ are preferentially followed by itself in the eukaryotic genes. This statistical result is observed with three ratios. Thus, the unitary circular codes of trinucleotides may have been involved in the formation of the trinucleotide circular code $X$. For the first time since 20 years, the circular code theory in genes is extended here to genomes. Circular code could be a mathematical structure of genes as well as genomes.

## Publications in Peer-Reviewed journals

Karim El Soufi and Christian J. Michel (2014). "Circular code motifs in the ribosome decoding center". In: Computational Biology and Chemistry 52, pp. 9-17. ISSN: 1476-9271. URL: http://www.sciencedirect.com/science/ article/pii/S1476927114000802

Karim El Soufi and Christian J. Michel (2015). "Circular code motifs near the ribosome decoding center". In: Computational Biology and Chemistry 59, Part A, pp. 158-176. ISSN: 1476-9271. URL: http : //www.sciencedirect.com/science/article/pii/S1476927115300335

Karim El Soufi and Christian J. Michel (2016). "Circular code motifs in genomes of eukaryotes". In: Journal of Theoretical Biology 408, pp. 198-212. ISSN: 0022-5193. URL: http://www.sciencedirect.com/science/article/ pii/S0022519316302053

Karim El Soufi and Christian J. Michel (2017). "Unitary circular code motifs in genomes of eukaryotes". In: BioSystems

## Bibliography

Anger, Andreas M. et al. (2013). "Structures of the human and Drosophila 80S ribosome". In: Nature 497.7447, pp. 80-85. ISSN: 0028-0836. URL: http: / / dx . doi.org/10.1038/nature12104.
Armache, Jean-Paul et al. (2010a). "Cryo-EM structure and rRNA model of a translating eukaryotic 80 S ribosome at $5.5-\AA$ resolution". In: Proceedings of the National Academy of Sciences 107.46, pp. 19748-19753. URL: http: //www.pnas.org/content/107/46/19748.abstract.
Armache, Jean-Paul et al. (2010b). "Localization of eukaryote-specific ribosomal proteins in a $5.5-\AA$ cryo-EM map of the 80 S eukaryotic ribosome". In: Proceedings of the National Academy of Sciences 107.46, pp. 19754-19759. URL: http://www.pnas.org/content/107/46/19754.abstract.
Armache, Jean-Paul et al. (2013). "Promiscuous behaviour of archaeal ribosomal proteins: Implications for eukaryotic ribosome evolution". In: Nucleic Acids Research 41.2, pp. 1284-1293. URL: http: / / nar. oxfordjournals.org / content/41/2/1284.abstract.
Arquès, D. G. and C. J. Michel (1996). "A complementary circular code in the protein coding genes." eng. In: J Theor Biol 182.1, pp. 45-58. DOI: 10.1006/ jtbi.1996.0142. URL: http://dx.doi.org/10.1006/jtbi.1996.0142.
Brilot, Axel F. et al. (2013). "Structure of the ribosome with elongation factor G trapped in the pretranslocation state". In: Proceedings of the National Academy of Sciences 110.52, pp. 20994-20999. URL: http: //www.pnas.org/content/ 110/52/20994.abstract.
Bulygin, Konstantin et al. (2009). "Sites of 18 S rRNA contacting mRNA 3 and 5 of the P site codon in human ribosome: A cross-linking study with mRNAs carrying 4-thiouridines at specific positions". In: Biochimica et Biophysica Acta (BBA) - Gene Regulatory Mechanisms 1789.3, pp. 167-174. ISSN: 18749399. URL: http://www. sciencedirect.com/science/article/pii/ S1874939908002691.
Chenna, Ramu et al. (2003). "Multiple sequence alignment with the Clustal series of programs". In: Nucleic Acids Research 31.13, pp. 3497-3500. url: http://nar.oxfordjournals.org/content/31/13/3497.abstract.

Crick, F H C, J S Griffith, and L E Orgel (1957). "Codes without commas". In: Proceedings of the National Academy of Sciences of the United States of America 43.5, pp. 416-421. ISSN: 1091-6490. URL: http://www.ncbi.nlm. nih.gov/pmc/articles/PMC528468/.
Demeshkina, Natalia et al. (2012). "A new understanding of the decoding principle on the ribosome". In: Nature 484.7393, pp. 256-259. ISSN: 0028-0836. URL: http://dx.doi.org/10.1038/nature10913.
El Soufi, Karim and Christian J. Michel (2016). "Circular code motifs in genomes of eukaryotes". In: Journal of Theoretical Biology 408, pp. 198-212. ISSN: 00225193. URL: http://www.sciencedirect.com/science/article/pii/ S0022519316302053.
El Soufi, Karim and Christian J. Michel (2014). "Circular code motifs in the ribosome decoding center". In: Computational Biology and Chemistry 52, pp. 917. ISSN: 1476-9271. URL: http://www. sciencedirect.com/science / article/pii/S1476927114000802.
El Soufi, Karim and Christian J. Michel (2015). "Circular code motifs near the ribosome decoding center". In: Computational Biology and Chemistry 59, Part A, pp. 158-176. ISSN: 1476-9271. URL: http://www.sciencedirect.com/ science/article/pii/S1476927115300335.
El Soufi, Karim and Christian J. Michel (2017). "Unitary circular code motifs in genomes of eukaryotes". In: BioSystems.
Fan-Minogue, Hua and David M Bedwell (2007). "Eukaryotic ribosomal RNA determinants of aminoglycoside resistance and their role in translational fidelity". In: RNA 14.1, pp. 148-157. ISSN: 1469-9001. URL: http: / /www.ncbi. nlm.nih.gov/pmc/articles/PMC2151042/.

Fimmel, Elena, Alberto Danielli, and Lutz Strüngmann (2013). "On dichotomic classes and bijections of the genetic code". In: Journal of Theoretical Biology 336, pp. 221-230. ISSN: 0022-5193. URL: http://www.sciencedirect.com/ science/article/pii/S0022519313003500.
Fimmel, Elena, Christian J. Michel, and Lutz Strüngmann (2016). "n-Nucleotide circular codes in graph theory". In: Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences 374.2063. ISSN: 1364-503X. DOI: 10.1098/rsta.2015.0058. eprint: http://rsta. royalsocietypublishing. org/content/374/2063/20150058. full. pdf. URL: http://rsta. royalsocietypublishing.org/content/374/ 2063/20150058.

Fimmel, Elena et al. (2014). "Circular codes, symmetries and transformations". In: Journal of Mathematical Biology 70.7, pp. 1623-1644. ISSN: 1432-1416. DOI: 10.1007/s00285-014-0806-7. URL: http://dx.doi.org/10.1007/ s00285-014-0806-7.
Gamow, G. (1954). "Possible Relation between Deoxyribonucleic Acid and Protein Structures". In: Nature 173.4398, pp. 318-318. URL: http: / / dx. doi . org/10.1038/173318a0.
Gogala, Marko et al. (2014). "Structures of the Sec61 complex engaged in nascent peptide translocation or membrane insertion". In: Nature 506.7486, pp. 107110. ISSN: 0028-0836. URL: http://dx.doi.org/10.1038/nature12950.

Hanson, Robert (2010). "Jmol - a paradigm shift in crystallographic visualization". In: J. Appl. Cryst. 43.5, pp. 1250-1260. ISSN: 0021-8898. URL: https: //doi.org/10.1107/S0021889810030256.
Herráez, Angel (2006). "Biomolecules in the computer: Jmol to the rescue". In: Biochem. Mol. Biol. Educ. 34.4, pp. 255-261. ISSN: 1539-3429. UrL: http: //dx.doi.org/10.1002/bmb.2006.494034042644.
Higgins, Desmond G, Julie D Thompson, and Toby J Gibson (1996). "Using CLUSTAL for multiple sequence alignments". In: Computer Methods for Macromolecular Sequence Analysis. Vol. Volume 266. Academic Press, pp. 383402. URL: http://www. sciencedirect. com/science/article / pii / S0076687996660248.
Jeanmougin, F et al. (1998). "Multiple sequence alignment with Clustal X". In: Trends in biochemical sciences 23.10, pp. 403-405. ISSN: 0968-0004. URL: http: //dx.doi.org/10.1016/S0968-0004(98)01285-7.
Jenner, Lasse B et al. (2010). "Structural aspects of messenger RNA reading frame maintenance by the ribosome". In: Nat Struct Mol Biol 17.5, pp. 555560. ISSN: 1545-9993. URL: http://dx.doi.org/10.1038/nsmb. 1790.

Larkin, M.A. et al. (2007). "Clustal W and Clustal X version 2.0". In: Bioinformatics 23.21, pp. 2947-2948. URL: http : / / bioinformatics . oxfordjournals.org/content/23/21/2947.abstract.
Michel, Christian J. (2014). "A genetic scale of reading frame coding". In: Journal of Theoretical Biology 355, pp. 83-94. ISSN: 0022-5193. URL: http://www . sciencedirect.com/science/article/pii/S0022519314001684.
Michel, Christian J. (2012). "Circular code motifs in transfer and 16S ribosomal RNAs: A possible translation code in genes". In: Computational Biology and Chemistry 37, pp. 24-37. ISSN: 1476-9271. URL: http://www. sciencedirect.com/science/article/pii/S147692711100096X.

Michel, Christian J. (2015). "The maximal C3 self-complementary trinucleotide circular code X in genes of bacteria, eukaryotes, plasmids and viruses". In: Journal of Theoretical Biology 380, pp. 156-177. ISSN: 00225193. URL: http://www.sciencedirect.com/science/article/pii/ S002251931500171X.
Michel, Christian J. and Giuseppe Pirillo (2010). "Identification of all trinucleotide circular codes". In: Computational Biology and Chemistry 34.2, pp. 122-125. ISSN: 1476-9271. URL: http://www. sciencedirect. com / science/article/pii/S1476927110000204.
Michel, Christian J., Giuseppe Pirillo, and Mario A. Pirillo (2012). "A classification of 20-trinucleotide circular codes". In: Information and Computation 212, pp. 55 -63. ISSN: 0890-5401. DOI: http://dx.doi.org/10. 1016/j.ic. 2011.12.001. URL: http://www.sciencedirect.com/science/article/ pii/S0890540111001702.
Michel, Christian J. and Hervé Seligmann (2014). "Bijective transformation circular codes and nucleotide exchanging RNA transcription". In: Biosystems 118, pp. 39-50. ISSN: 0303-2647. URL: http://www. sciencedirect. com/ science/article/pii/S0303264714000215.
Moazed, D. and H. F. Noller (1990). "Binding of tRNA to the ribosomal A and P sites protects two distinct sets of nucleotides in 16 S rRNA." eng. In: $J$ Mol Biol 211.1, pp. 135-145. DOI: 10.1016/0022-2836(90) 90016-F. URL: http://dx.doi.org/10.1016/0022-2836(90)90016-F.
Nirenberg, Marshall W. and J. Heinrich Matthaei (1961). "The dependence of cell-free protein synthesis in E. coli upon naturally occurring or synthetic polyribonucleotides". In: Proceedings of the National Academy of Sciences 47.10, pp. 1588-1602. URL: http : / / www. pnas. org / content / 47 / 10 / 1588. short.

Powers, T. and H. F. Noller (1994). "Selective perturbation of G530 of 16 S rRNA by translational miscoding agents and a streptomycin-dependence mutation in protein S12." eng. In: J Mol Biol 235.1, pp. 156-172.
Reuveni, Shlomi et al. (2011). "Genome-Scale Analysis of Translation Elongation with a Ribosome Flow Model". In: PLoS Computational Biology 7.9, e1002127. ISSN: 1553-7358. URL: http://www.ncbi.nlm.nih.gov/pmc/articles/ PMC3164701/.
Seligmann, Hervé (2013a). "Polymerization of non-complementary RNA: Systematic symmetric nucleotide exchanges mainly involving uracil produce mitochondrial RNA transcripts coding for cryptic overlapping genes". In:

Biosystems 111.3, pp. 156-174. ISSN: 0303-2647. URL: http : / / www . sciencedirect.com/science/article/pii/S0303264713000269.
Seligmann, Hervé (2013b). "Systematic asymmetric nucleotide exchanges produce human mitochondrial RNAs cryptically encoding for overlapping protein coding genes". In: Journal of Theoretical Biology 324, pp. 1-20. ISSN: 00225193. URL: http://www.sciencedirect.com/science/article/pii/ S0022519313000490.
Sharma, Manjuli R. et al. (2007). "Cryo-EM study of the spinach chloroplast ribosome reveals the structural and functional roles of plastid-specific ribosomal proteins". In: Proceedings of the National Academy of Sciences 104.49, pp. 19315-19320. URL: http://www. pnas.org/content/104/49/19315. abstract.
Thompson, JD et al. (1997). "The CLUSTAL X windows interface: flexible strategies for multiple sequence alignment aided by quality analysis tools". In: Nucleic acids research 25.24, pp. 4876-4882. ISSN: 0305-1048. URL: http: //dx.doi.org/10.1093/nar/25.24.4876.
Watson, J. D. and F. H. C. Crick (1953). "Molecular Structure of Nucleic Acids: A Structure for Deoxyribose Nucleic Acid". In: Nature 171.4356, pp. 737-738. URL: http://dx.doi.org/10.1038/171737a0.
Willighagen, Egon L (2001). "Processing CML conventions in Java". In: Internet Journal of Chemistry 4.4, pp. 1099-8292.
Wilson, Daniel N. (2014). "Ribosome-targeting antibiotics and mechanisms of bacterial resistance". In: Nat Rev Micro 12.1, pp. 35-48. ISSN: 1740-1526. URL: http://dx.doi.org/10.1038/nrmicro3155.
Yoshizawa, Satoko, Dominique Fourmy, and Joseph D. Puglisi (1999). "Recognition of the Codon-Anticodon Helix by Ribosomal RNA". In: Science 285.5434, pp. 1722-1725. URL: http: / /science.sciencemag.org/content/285/ 5434/1722.abstract.

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## Study of circular code motifs in nucleic acid sequences

Le travail effectué dans cette thèse présente une nouvelle approche de la théorie du code circulaire dans les gènes qui a été initiée en 1996. Cette approche consiste à analyser les motifs construits a partir de ce code circulaire, ces motifs particulier sont appelés motifs de code circulaire. Ainsi, nous avons développé des algorithmes de recherche pour localiser les motifs de code circulaire dans les séquences d'acides nucleiques afin de leur trouver une signification bioinformatique. En effet, le code circulaire $X$ identifie dans les gènes est un ensemble de trinucleotides qui a la propriété de retrouver, synchroniser et maintenir la phase de lecture. Nous avons commence notre analyse avec le centre de décodage du ribosome ( ARNr ) qui est une région majeure dans le processus de traduction des gènes aux protéines. Puis, nous avons étendu les résultats obtenus avec le ribosome aux ARN de transfert (ARNt) pour étudier les interactions ARNr-ARNt. Enfin, nous avons généralisé la recherche de motifs de code circulaire $X$ dans l'ADN aux chromosomes d'eucaryotes complets.

The work done in this thesis presents a new direction for circular code identified in 1996 by analysing the motifs constructed from circular code. These particular motifs are called circular code motifs. We applied search algorithms to locate circular code motifs in nucleic acid sequences in order to find biological significance. In fact, the circular code X , which was found in gene sequences, is a set of trinucleotides that have the property of reading frame retrieval, synchronization and maintenance. We started our study in the ribosomal decoding centre (rRNA), an important region involved in the process of translating genes into proteins. Afterwards, we expanded our scope to study the interaction of rRNA through the $X$ circular code. Finally, we search for the $X$ circular code motifs in the complete DNA sequences of chromosomes of the eukaryotic genomes. This study introduced new properties to the circular code theory.

